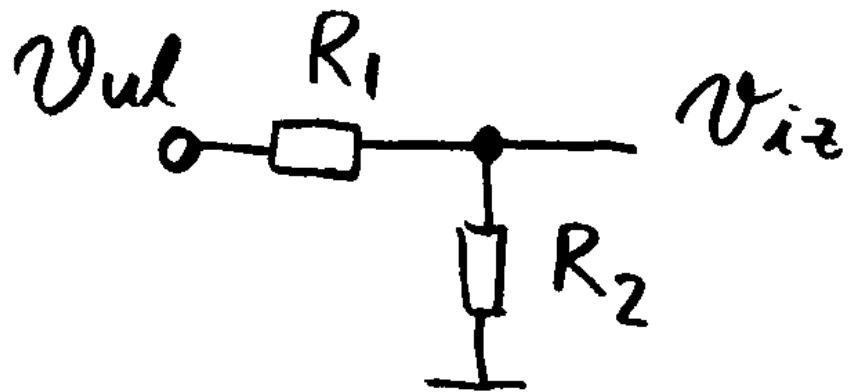


# Frekvencijske karakteristike elektronskih kola

Prenosne funkcije (nule i polovi)  
Amplitudno-frekvencijski i  
fazno-frekvencijski dijagrami

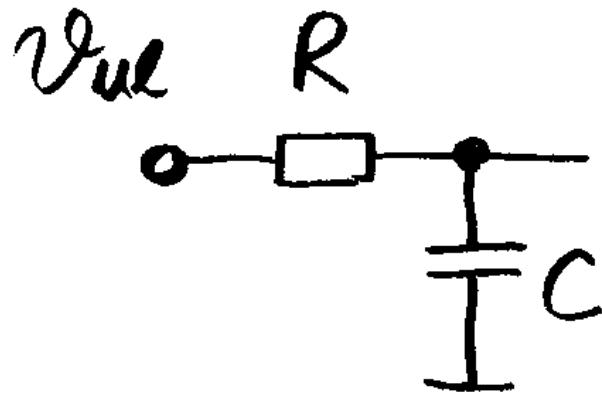
Frekvenčijski nezavisno kolo  
Pojacanje A ne zavisi od f



$$V_{iz} = \frac{R_2}{R_1 + R_2} V_{ul}$$

$$A = \frac{V_{iz}}{V_{ul}} = \frac{R_2}{R_1 + R_2}$$

# Frekvencijski zavisno kolo primjer 1.



$$A = \frac{V_{iz}(s)}{V_{ul}(s)} = \frac{\frac{1}{sc}}{R + \frac{1}{sc}}$$

$$A = \frac{1}{1 + SCR}$$

$$Sp = -\frac{1}{CR}$$

Ucestanost (frekvencija) s

Nulta ucestanost  $s=0$

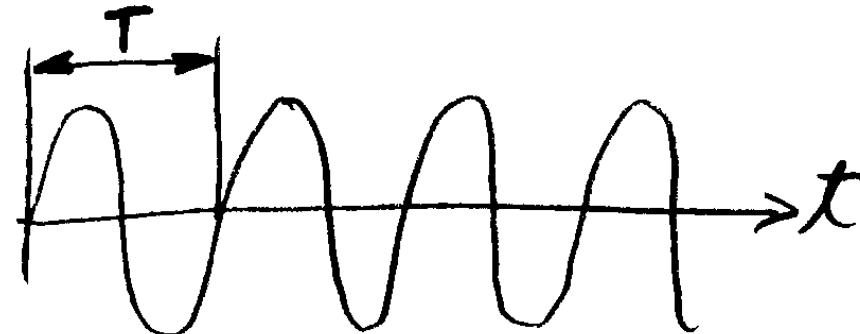
Realna ucestanost  $s=j\omega$

$$s = 0$$



јерносијерни сигнал

$$s = j\omega$$



хармонички сигнал  
(сингулярен сигнал)

$\omega$  = крутина  
услонености

$$\omega = 2\pi f$$

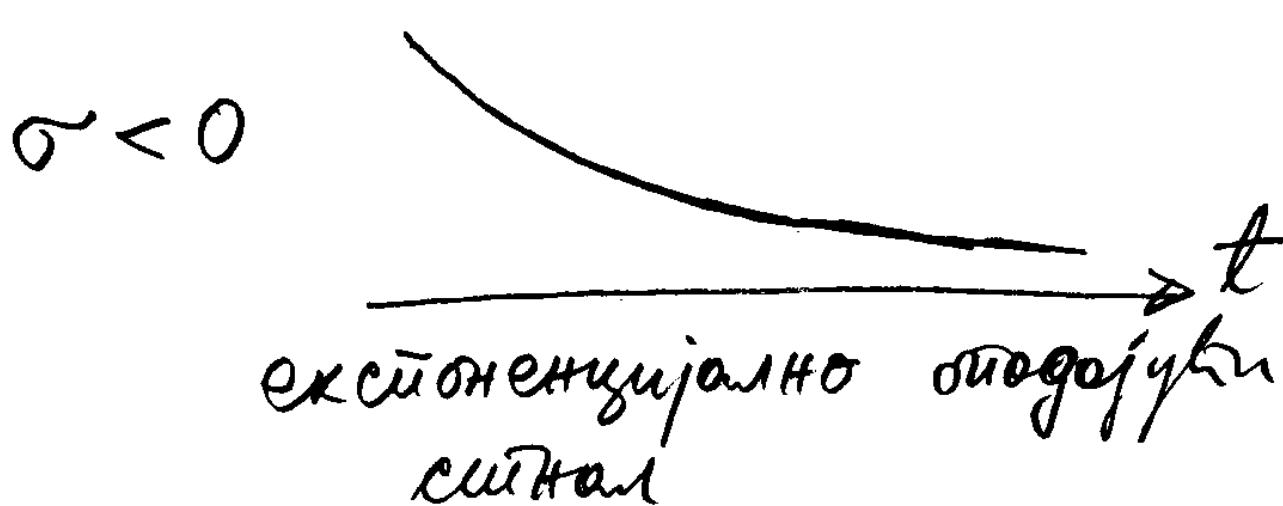
$$f = \frac{1}{T}$$

$$T = \text{период}$$

# Imaginarna ucestanost

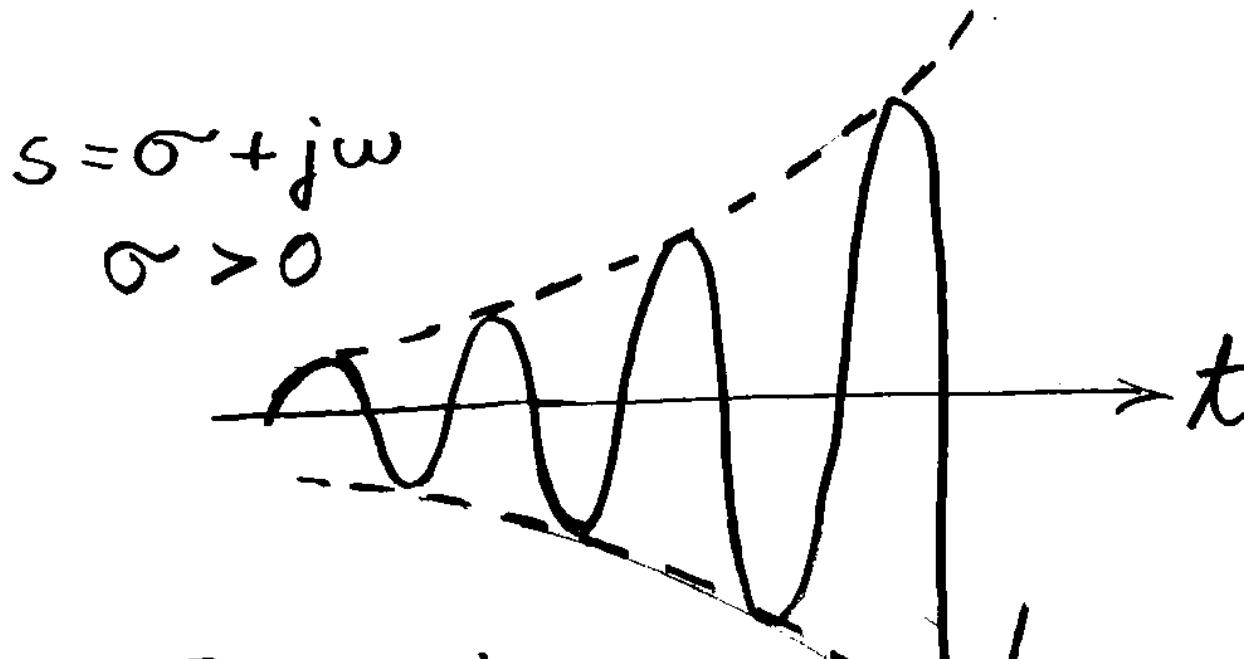


експоненцијално растући  
сигнал



експоненцијално спадајући  
сигнал

# Kompleksna ucestanost



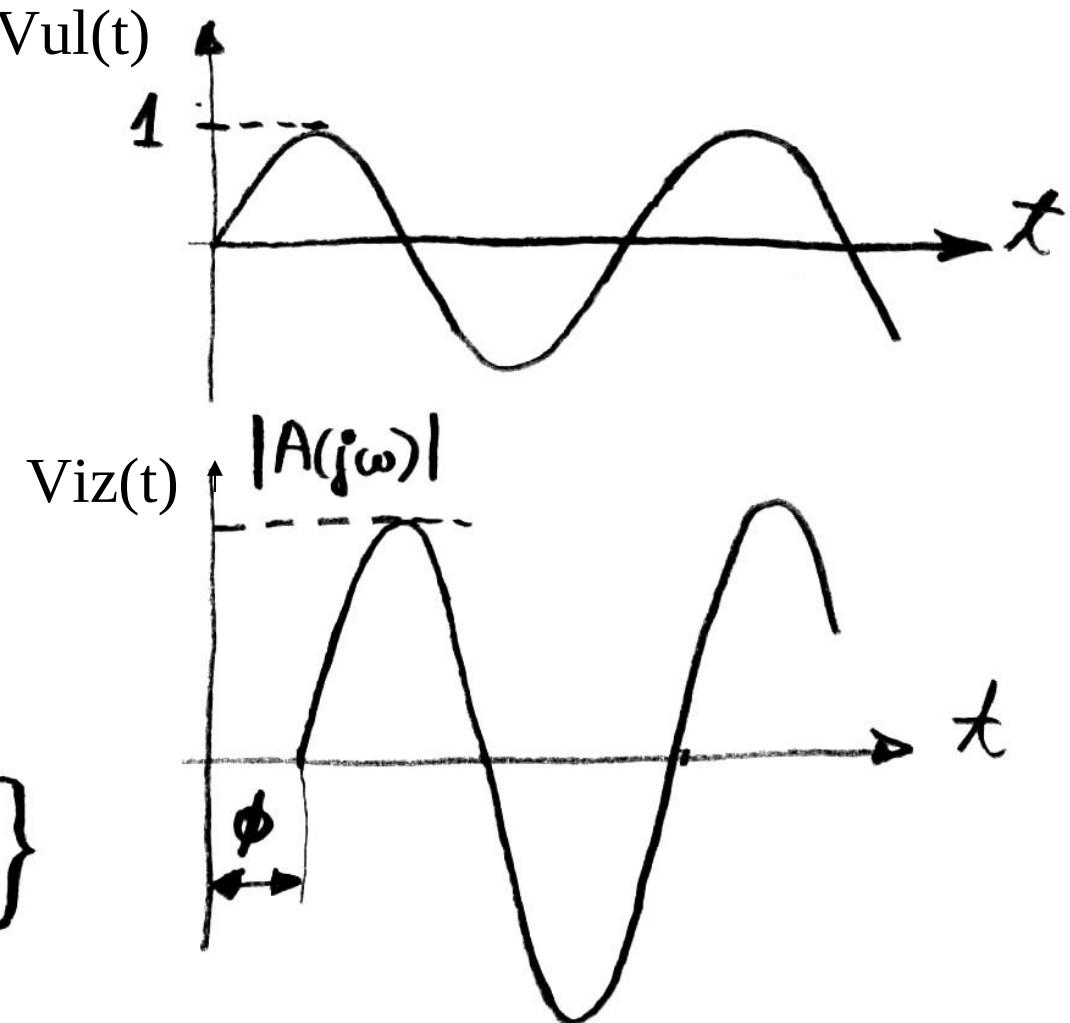
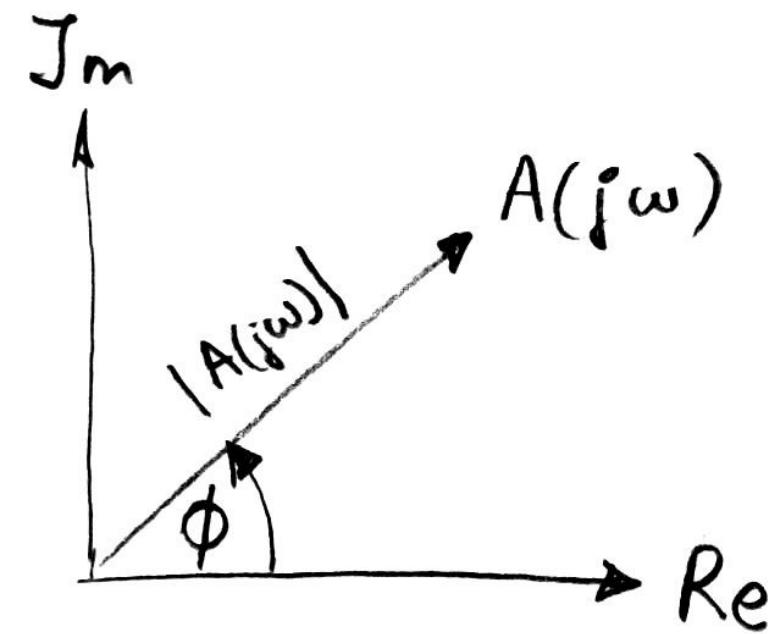
експоненцијално  
распушти хармонички  
сигнал

$\omega$  = реална честотност

$\sigma$  = имагинарна честотност

Kada u  $A(s)$  stavimo  
 $s=jw$ , dobijamo  
prenosnu funkciju za  
realne ucestanosti  $A(jw)$

# Kompleksno pojicanje $A(j\omega)$



$$\phi = \text{Arg} \{ A(j\omega) \}$$

# Moduo pojacanja - primjer 1

$$|A(j\omega)| = \left| \frac{1}{1 + j\omega CR} \right| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

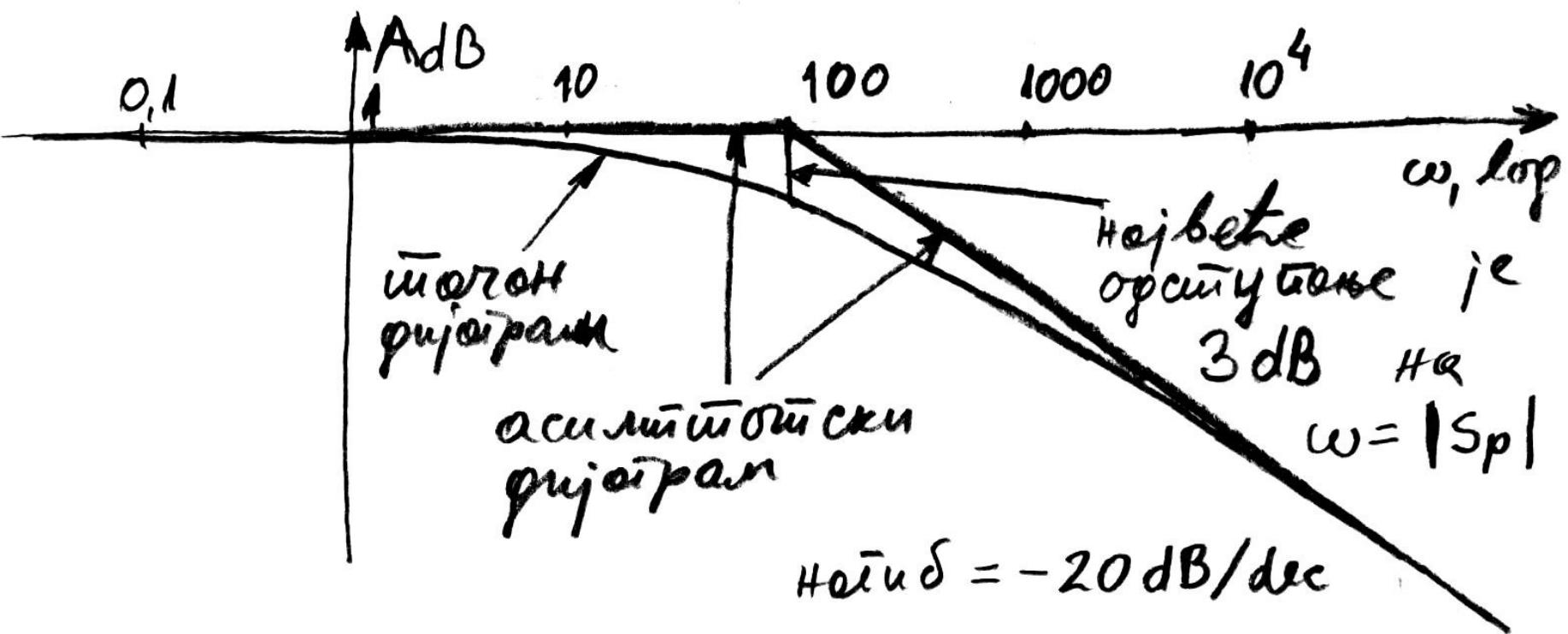
$$A_{dB} = 20 \log |A(j\omega)| = 20 \log \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

$$A_{dB} = 20 (-1) \frac{1}{2} \log (1 + \omega^2 C^2 R^2)$$

# Asimptote AdB linearna f(logw)

$$\omega \rightarrow 0 \quad \text{AdB} \rightarrow -10 \log 1 = 0$$

$$\omega CR \gg 1 \quad \text{AdB} = -10 \log(\omega^2 C^2 R^2) = -20 \log \omega |CR|$$



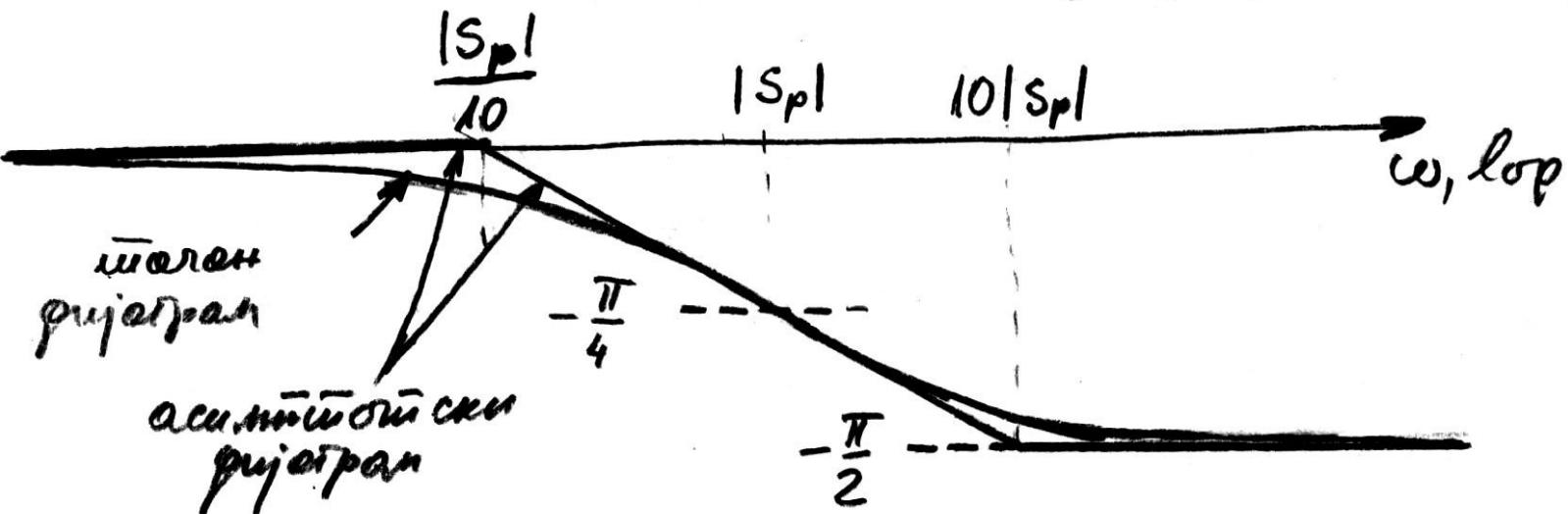
# Logaritamsko pojanje dB

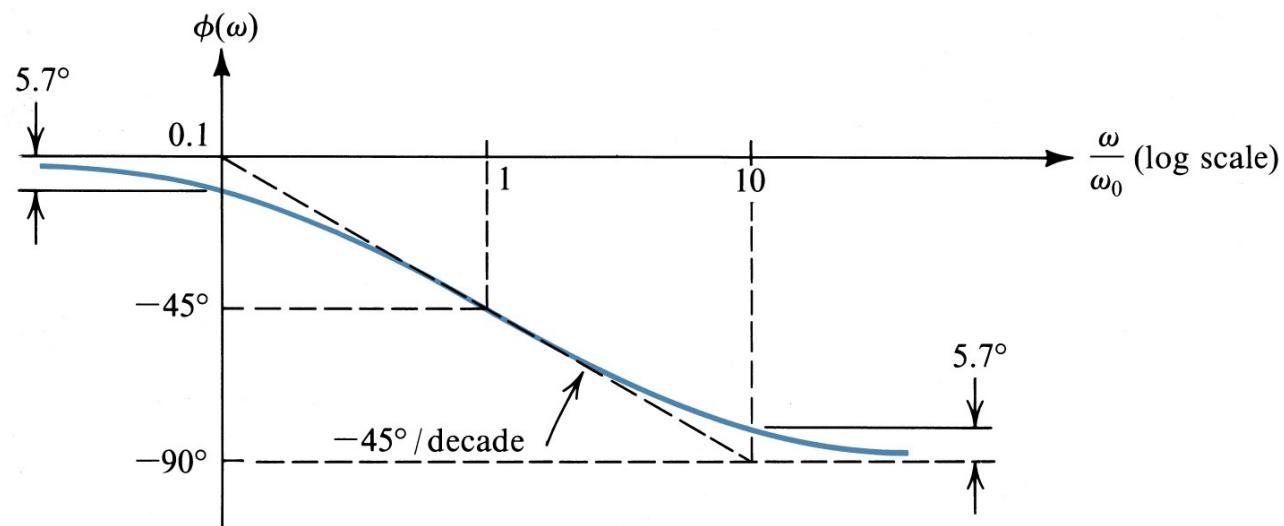
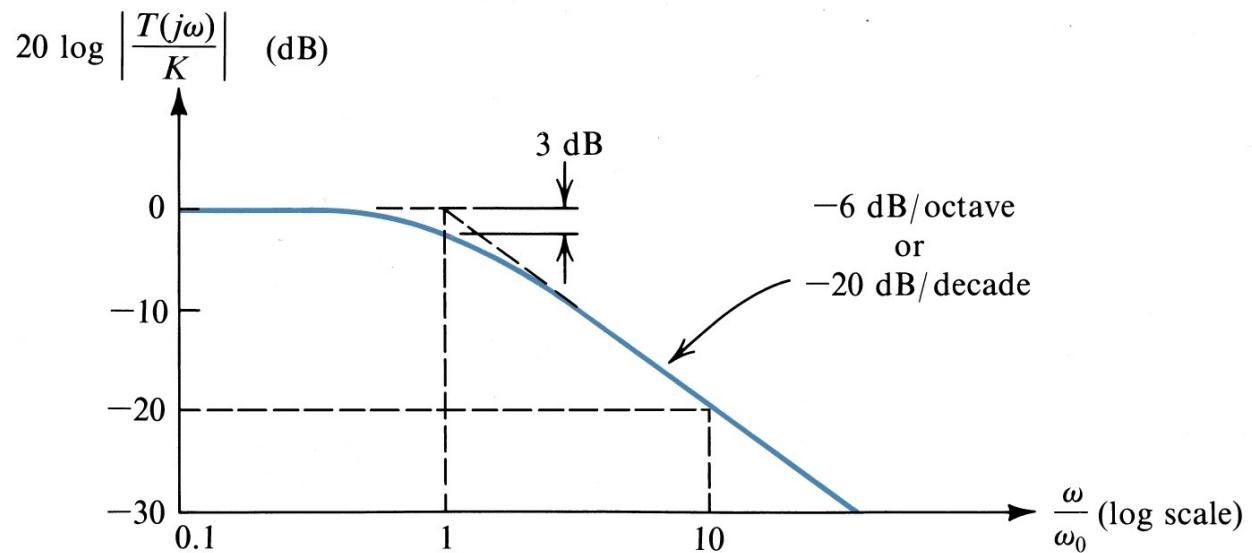
- $A = 0.001 \quad \text{dB} = -60\text{dB}$
- $A = 0.01 \quad \text{dB} = -40\text{dB}$
- $A = 0.1 \quad \text{dB} = -20\text{dB}$
- $A = 1/2 \quad \text{dB} = -6\text{dB}$
- $A = 1 \quad \text{dB} = 0\text{dB}$
- $A = 2 \quad \text{dB} = +6\text{dB}$
- $A = 10 \quad \text{dB} = +20\text{dB}$
- $A = 100 \quad \text{dB} = +40\text{dB}$

# Faza prenosne funkcije - primjer 1

$$A(j\omega) = \frac{1}{1 + j\omega CR}$$

$$\phi(j\omega) = \text{Arg}\{A(j\omega)\} = \arctg \frac{\text{Im}\{A(j\omega)\}}{\text{Re}\{A(j\omega)\}} = -\arctg \frac{\omega RC}{1}$$





**Fig. 1.23 (a)** Magnitude and **(b)** phase response of STC networks of the low-pass type.

# Prenosna funkcija sa nulom

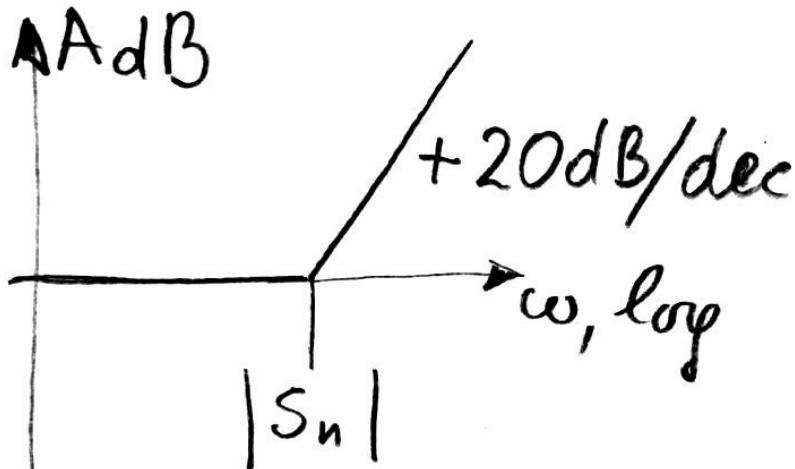
$$A(s) = 1 + sRC \quad s_n = -\frac{1}{RC}$$

$$|A(j\omega)| = \sqrt{1 + \omega^2 R^2 C^2}$$

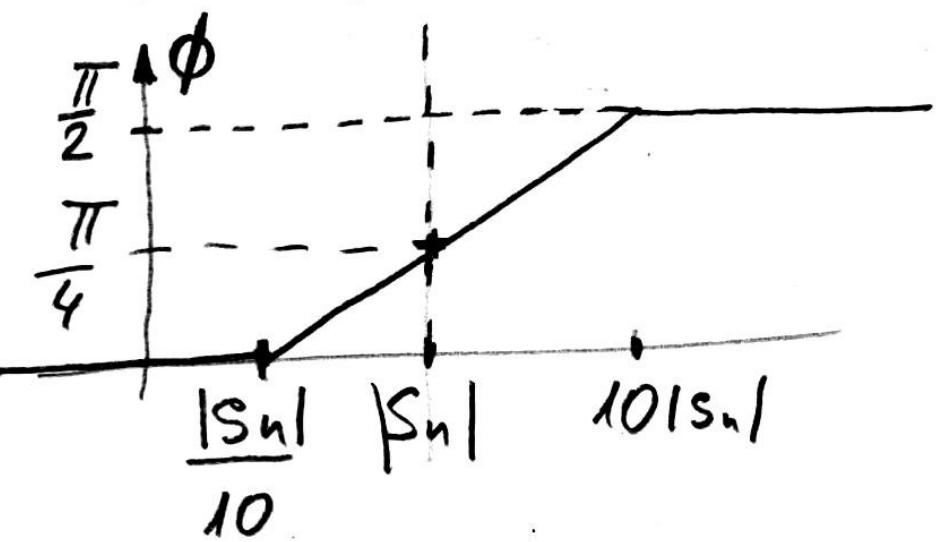
$$A_{dB} = 20 \log |A(j\omega)| = +10 \log (1 + \omega^2 R^2 C^2)$$

$$\phi = \text{Arg}\{A(j\omega)\} = \arctg \omega RC$$

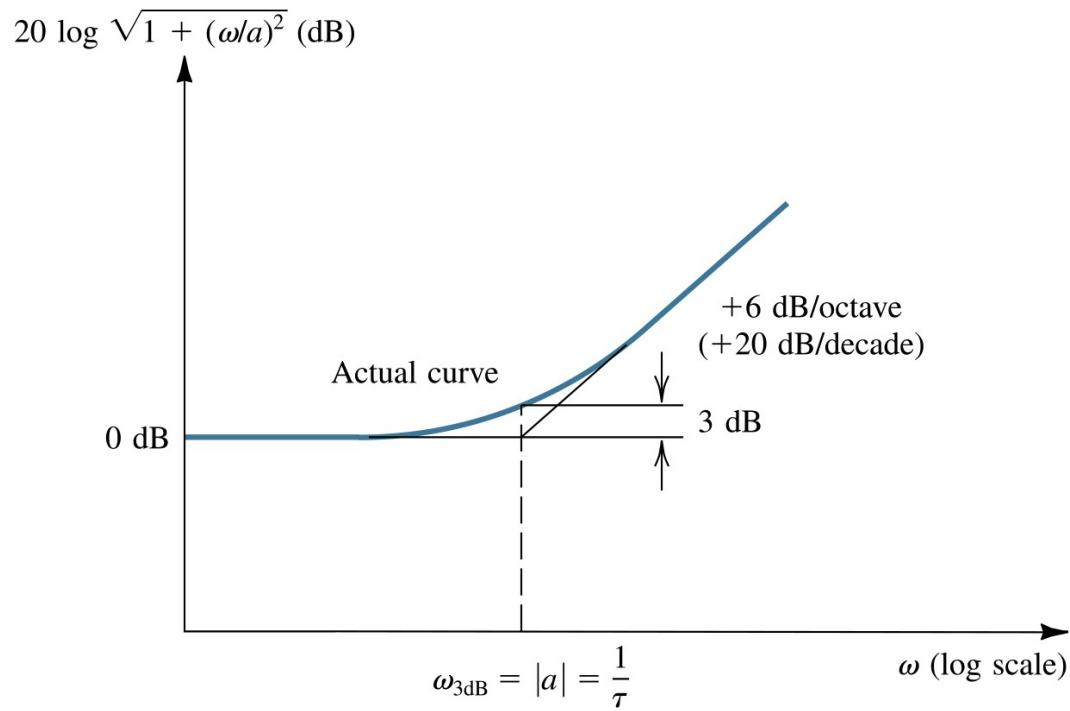
# Amplitudski i fazni dijagram prenosne funkcije sa nulom



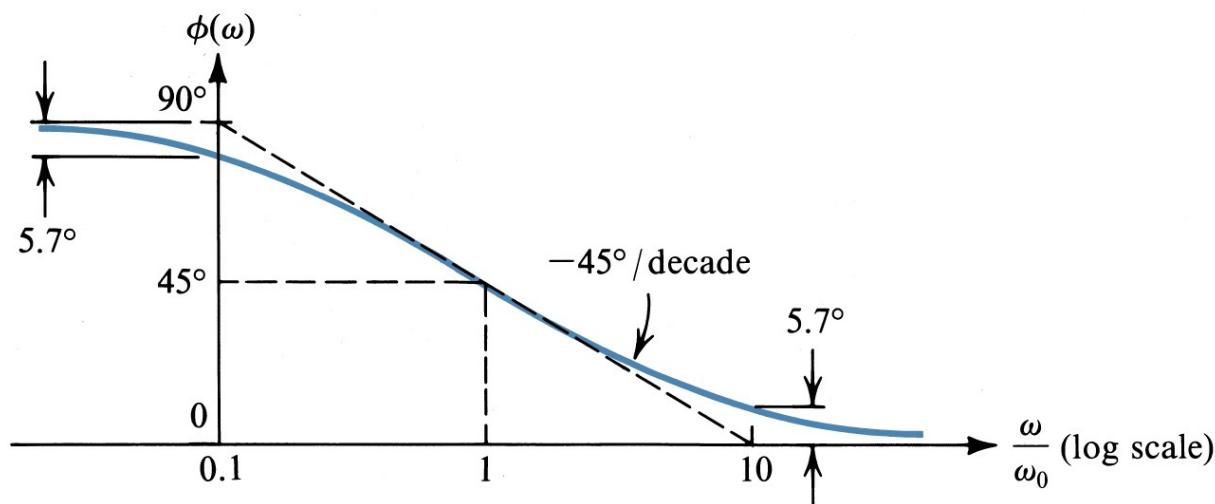
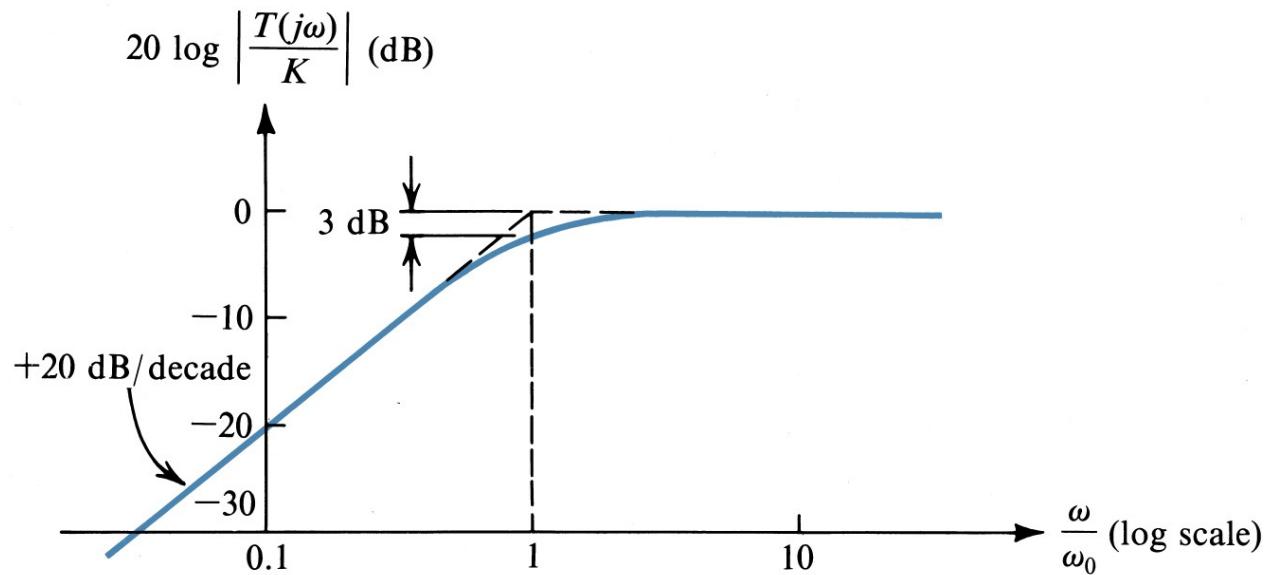
Нула јављаје  
измену  $A_{dB}$  за  
 $20dB/dec$



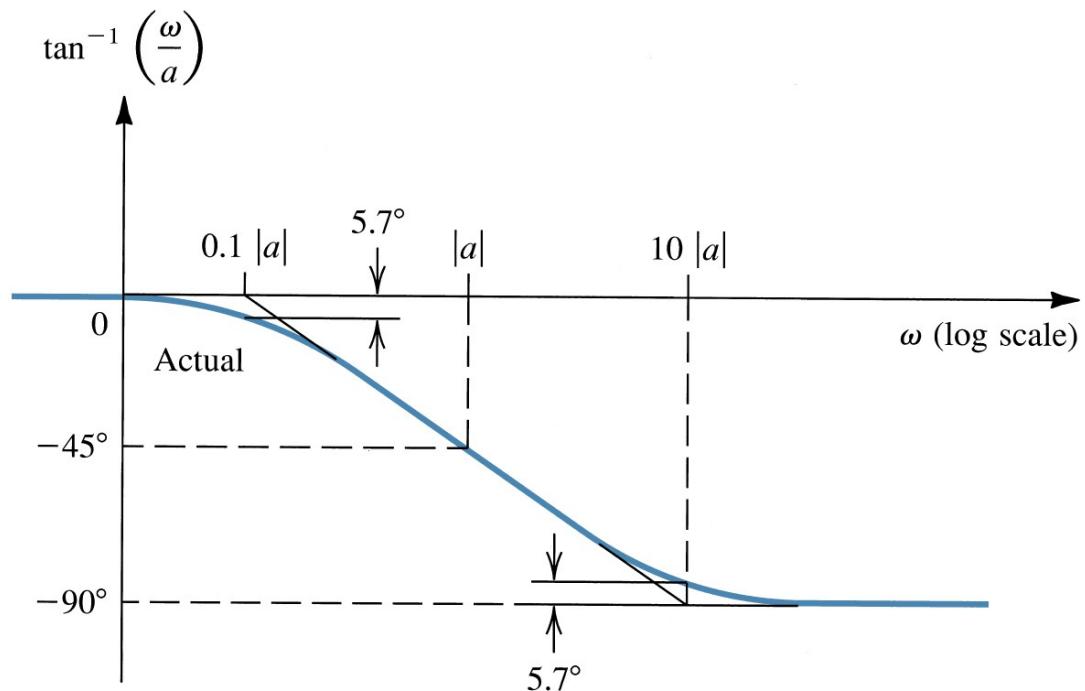
Нула јављаје  
поменуту  
фазну корактност ук  
зе  $\frac{\pi}{2}$ .



**Fig. 7.1** Bode plot for the typical magnitude term. The curve shown applies for the case of a zero. For a pole, the high-frequency asymptote should be drawn with a  $-6\text{-dB/octave}$  slope.



**Fig. 1.24 (a)** Magnitude and **(b)** phase response of STC networks of the high-pass type.



**Fig. 7.3** Bode plot of the typical phase term  $\tan^{-1} (\omega/a)$  when  $a$  is negative.

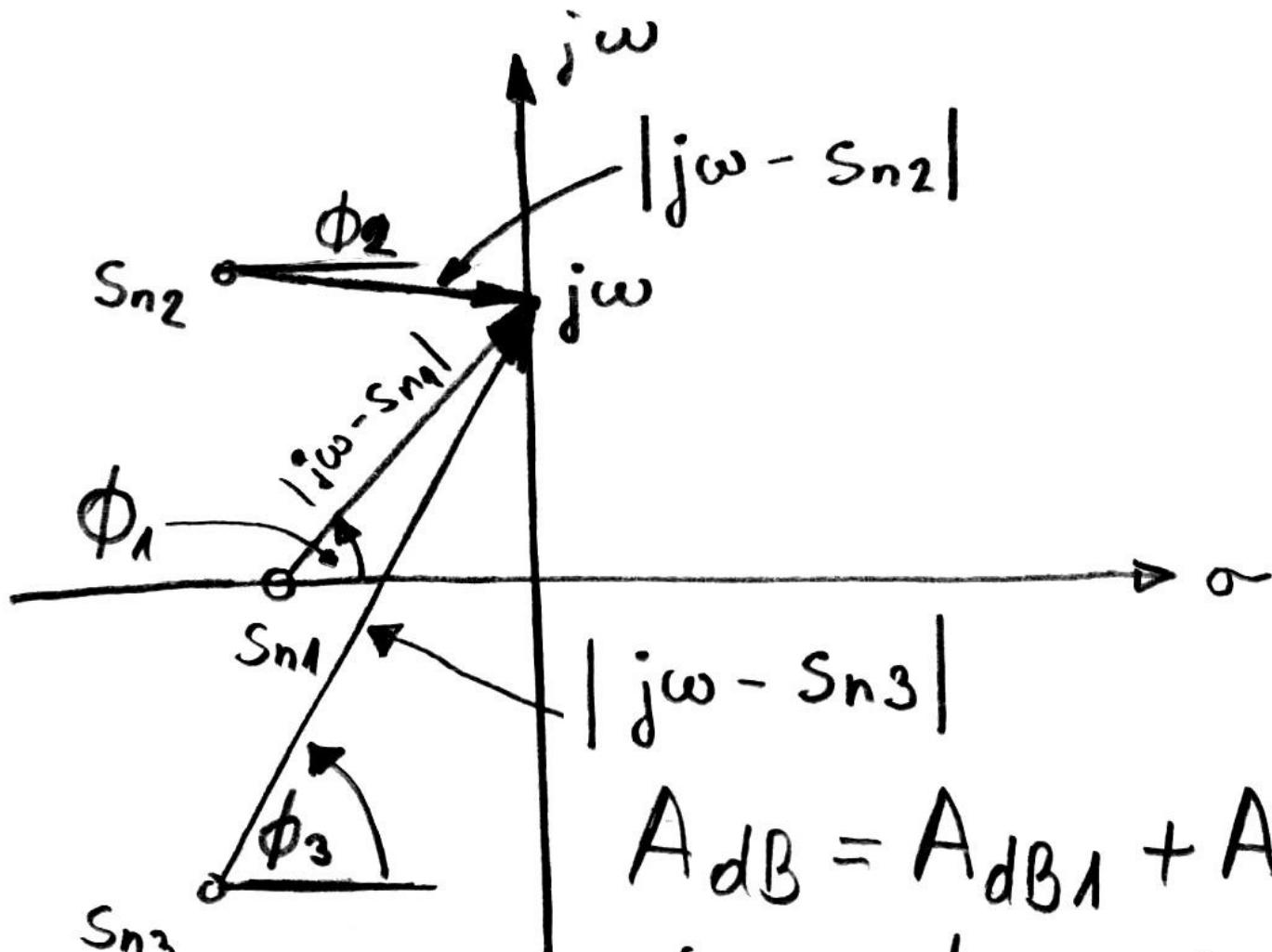
U opstem slucaju kolo ima vise  
nula i polova

$$A(j\omega) = K \frac{(s - s_{n_1})(s - s_{n_2}) \dots}{(s - s_{p_1})(s - s_{p_2}) \dots}$$

$$\begin{aligned} A_{dB} = & 20 \log k + 20 \log |j\omega - s_{n_1}| + 20 \log |j\omega - s_{n_2}| + \dots \\ & - 20 \log |j\omega - s_{p_1}| - 20 \log |j\omega - s_{p_2}| - \dots \end{aligned}$$

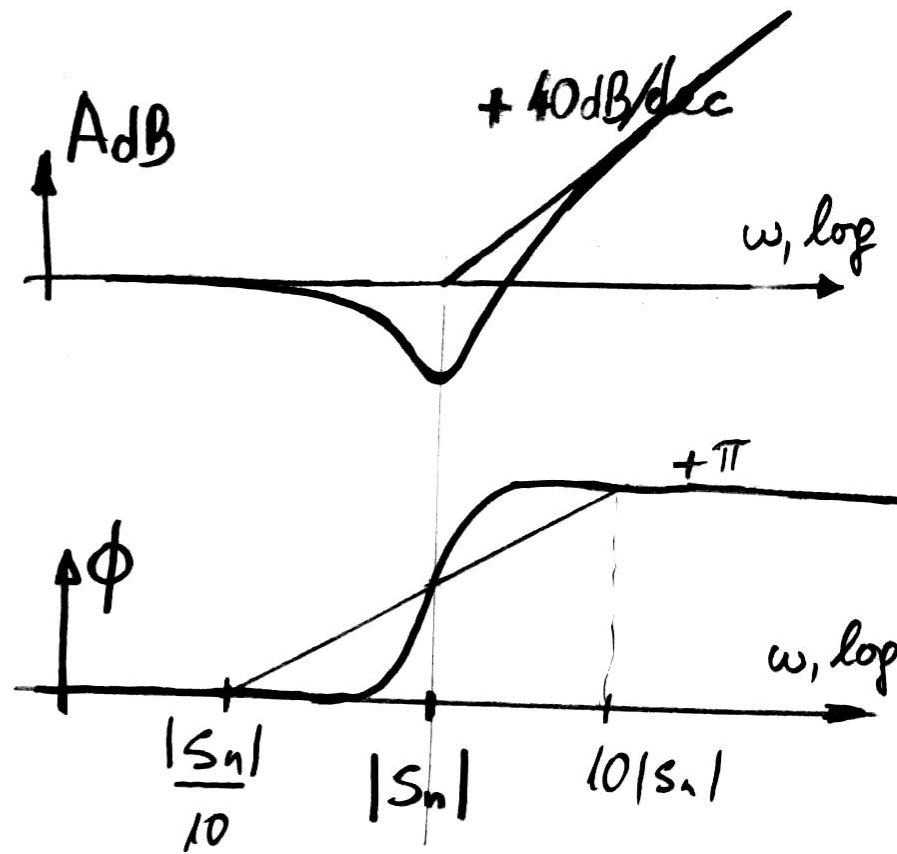
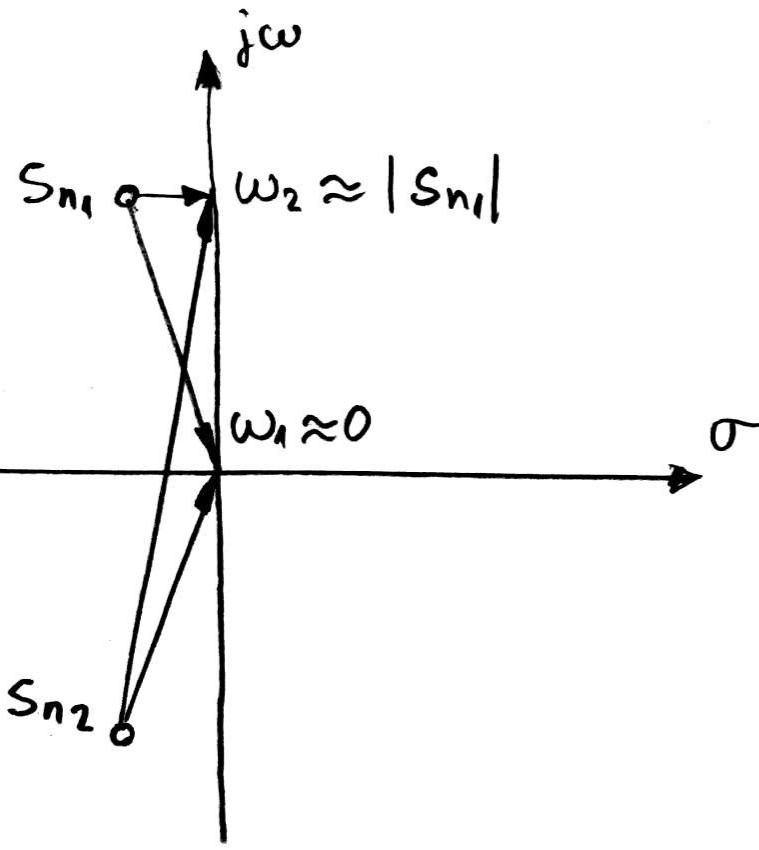
$$\begin{aligned} \phi = & \operatorname{Arg} \{ j\omega - s_{n_1} \} + \operatorname{Arg} \{ j\omega - s_{n_2} \} + \dots \\ & - \operatorname{Arg} \{ j\omega - s_{p_1} \} - \operatorname{Arg} \{ j\omega - s_{p_2} \} - \dots \end{aligned}$$

# Uticaj nula na AFK i FFK

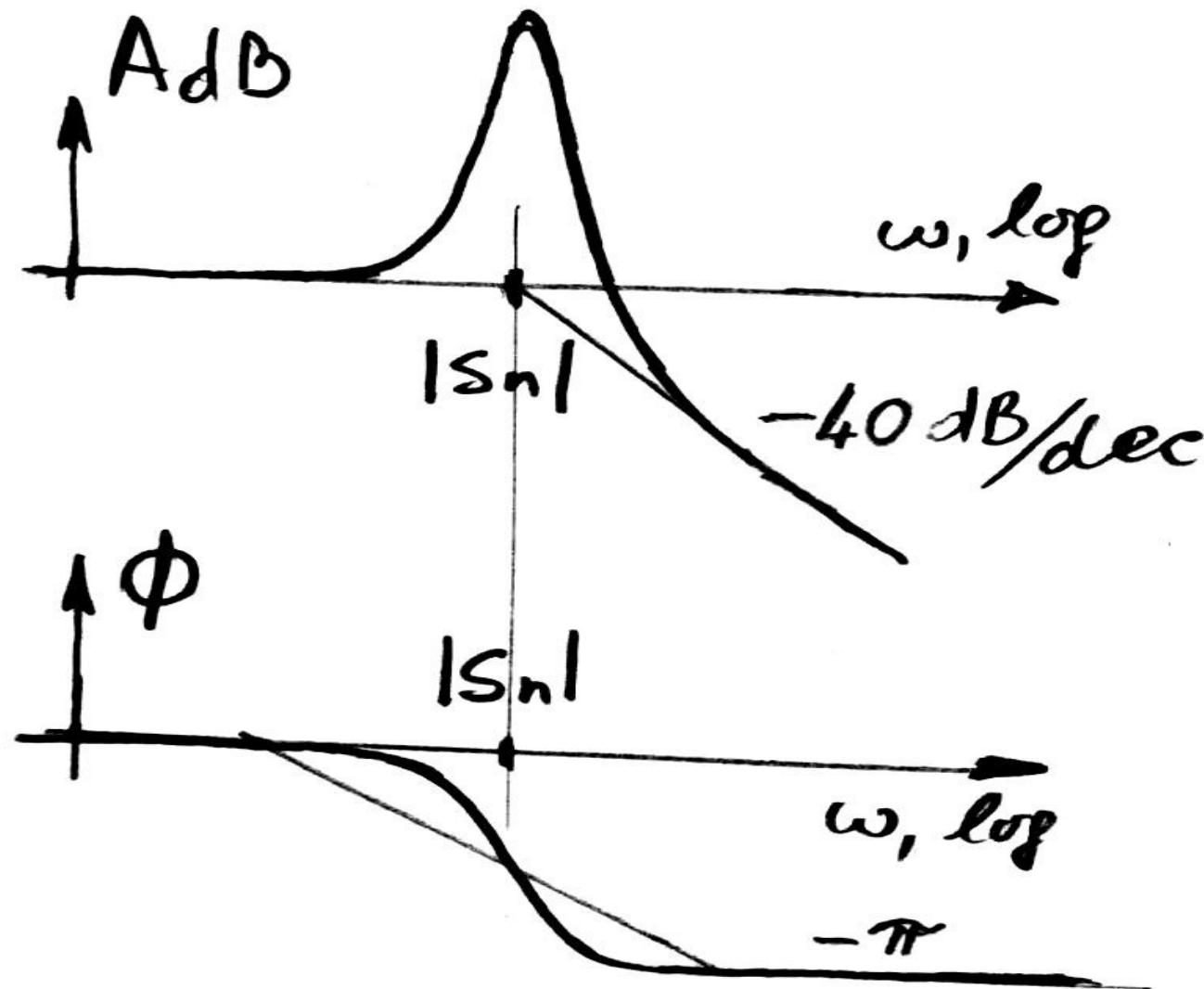


$$\phi = \phi_1 + \phi_2 + \phi_3$$

Konjugovano kompleksni par nula blizu  $j\omega$  ose unosi rezonantno ulegnuće i strmiju promjenu faze



# Uticaj slabo prigusenih polova na AFK i FFK



# Analiza frekvencijskih karakteristika

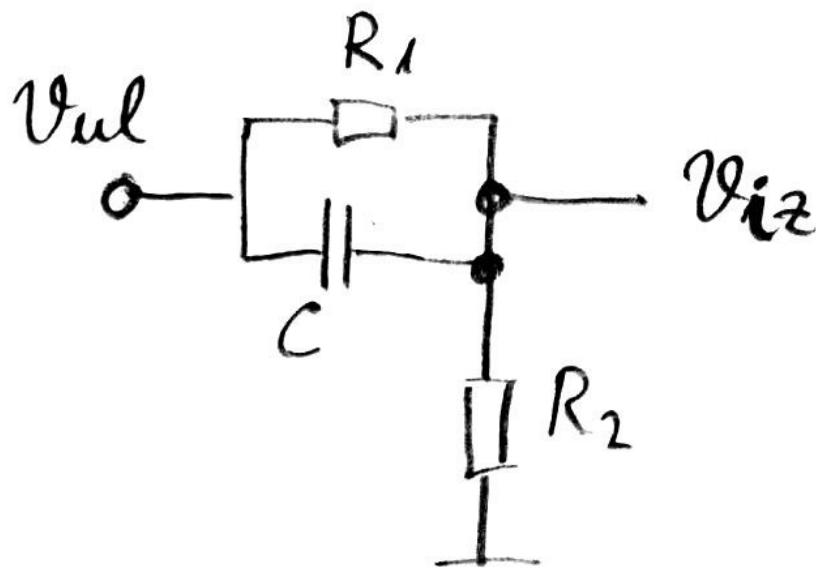
- Nalazenje prenosne funkcije kola
- Nalazenje nula i polova ove funkcije
- Crtanje AF i FF dijagrama

# Nalazenje prenosne funkcije kola

- Elektronske komponente (tranzistore, diode, itd) zamjenimo modelima za male signale i svodimo problem na linearno kolo sa koncentrisanim parametrima.
- Reaktanse uvijek donose polove.

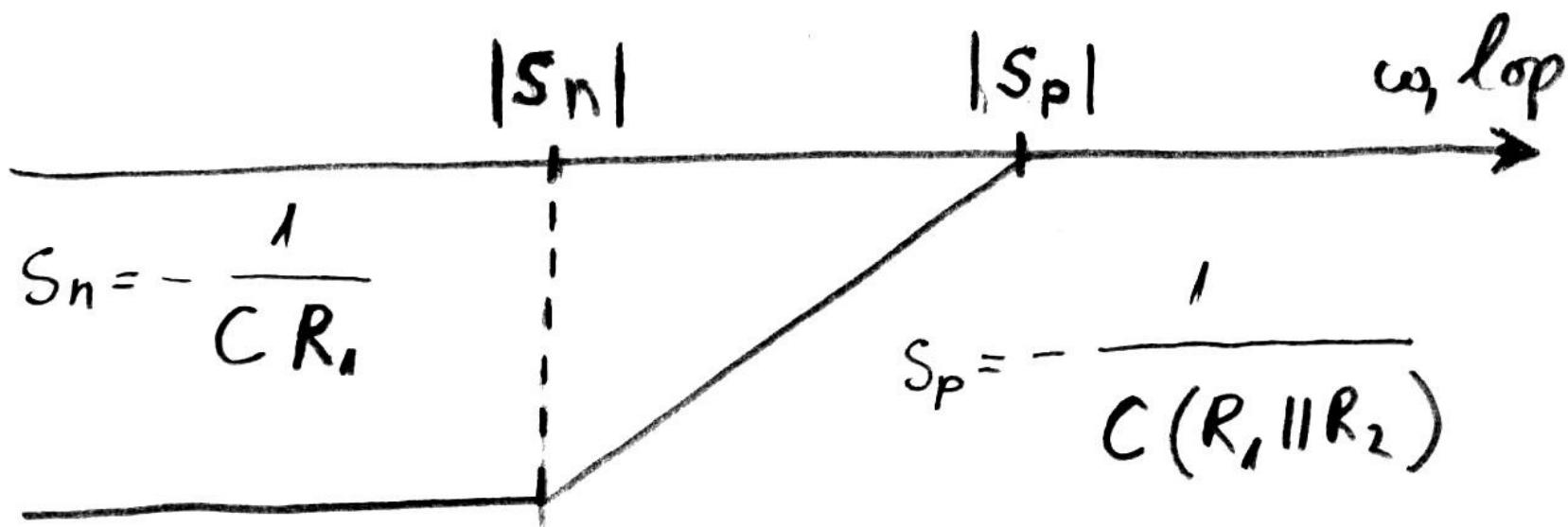


- Kondenzator u direktnoj grani donosi nulu.

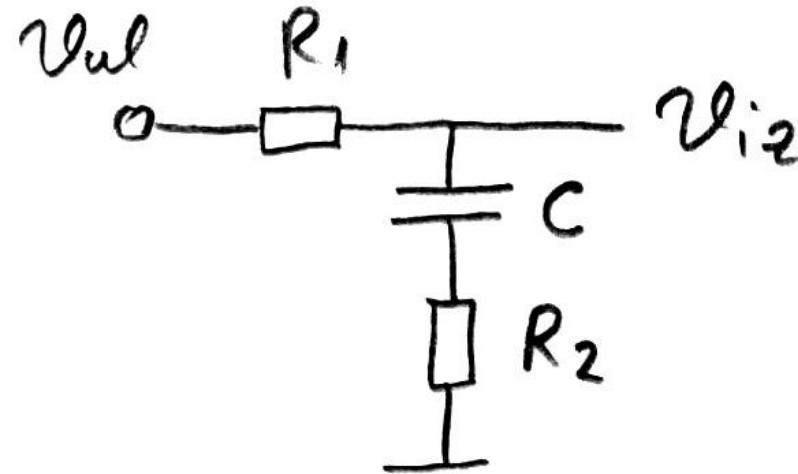


$$\omega = 0 \Rightarrow A = \frac{R_2}{R_1 + R_2}$$

$$\omega \rightarrow \infty \Rightarrow A = 1$$

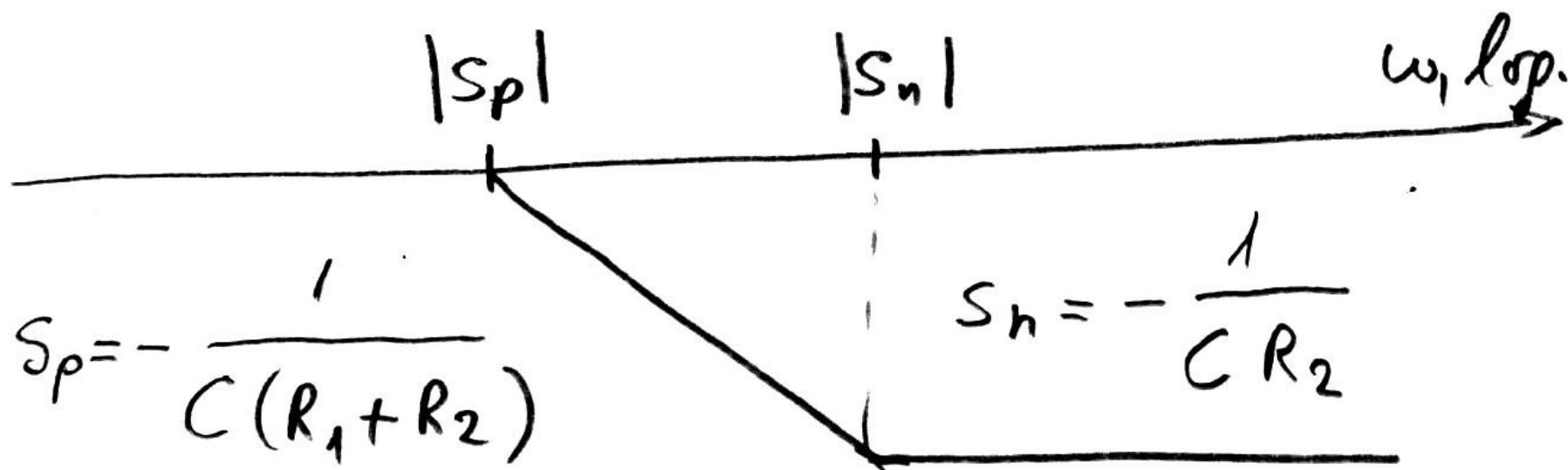


- Kondenzator u otocnoj grani donosi nulu kada je vezan na red sa otpornikom.



$$\omega = 0 \quad A = 1$$

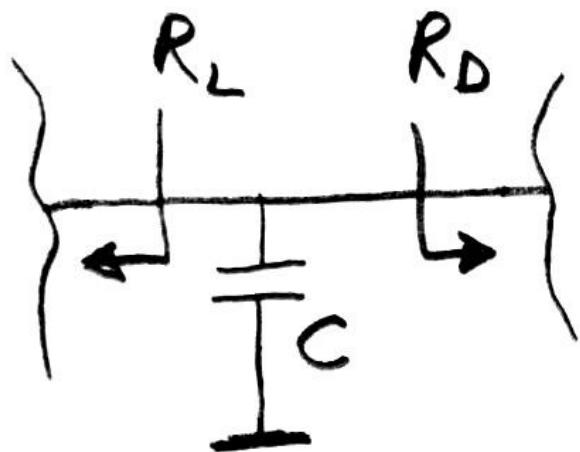
$$\omega \rightarrow \infty \quad A = \frac{R_2}{R_1 + R_2}$$



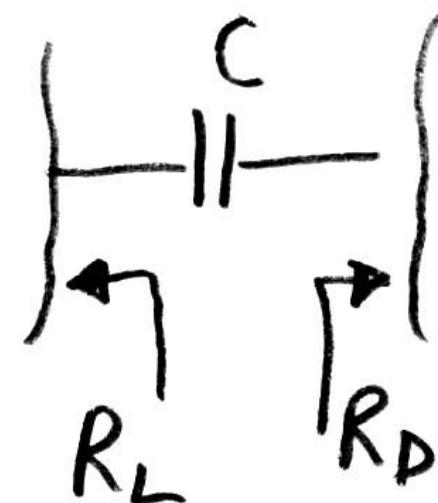
- Induktivnost u otocnoj grani donosi nulu.
- Induktivnost u direktnoj grani donosi nulu kada je na red vezana sa otpornikom.
- Ucestanost pola  $S_p = -1/t$ , gdje je  $t = C^*R_e$ , gdje je  $R_e$  ekvivalentna otpornost koju “vidi” kondenzator.
- Analogno  $t = L/R_e$ , gdje je  $R_e$  otpornost koju “vidi” induktivitet.

# Nalazenje pola od kondenzatora

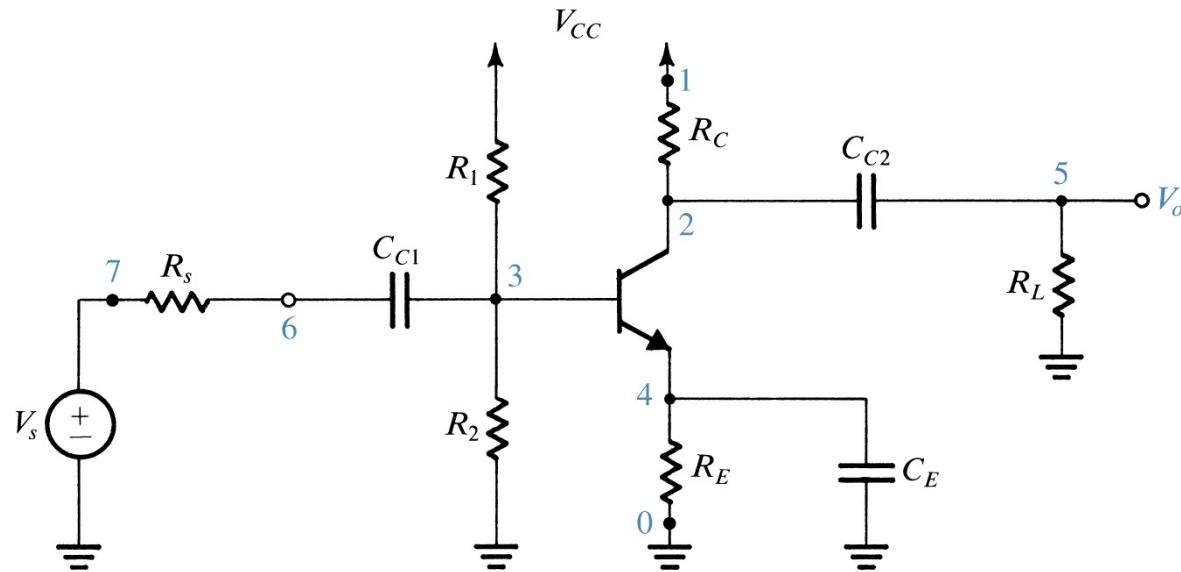
$$S_p = -\frac{1}{\tilde{\tau}} \quad \tilde{\tau} = C \cdot R_{ek}$$



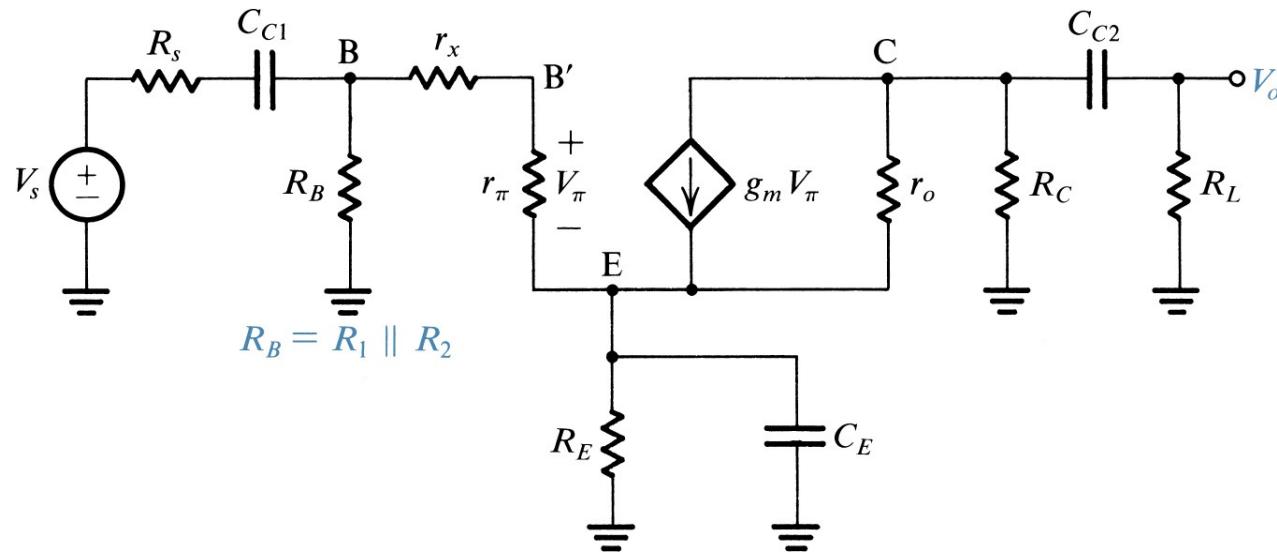
$$\tilde{\tau} = C \cdot (R_L \parallel R_D)$$



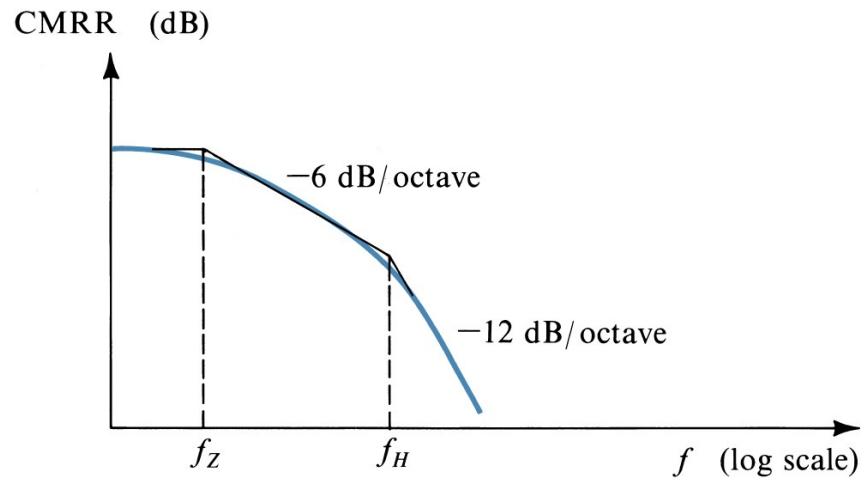
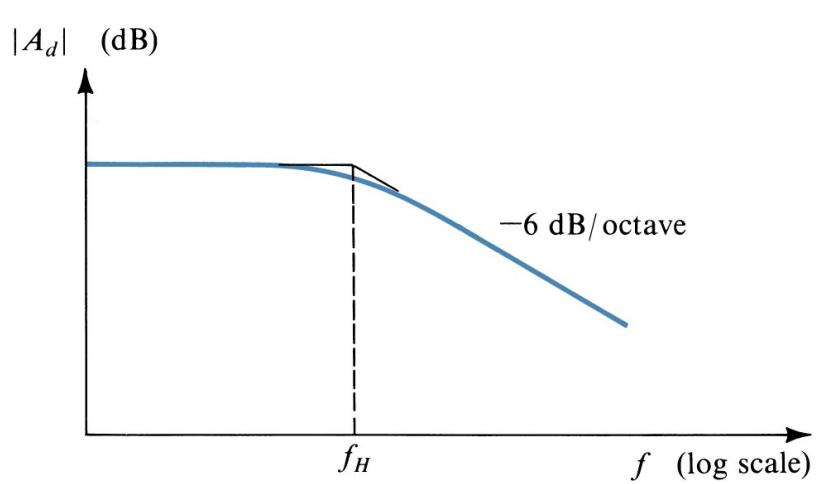
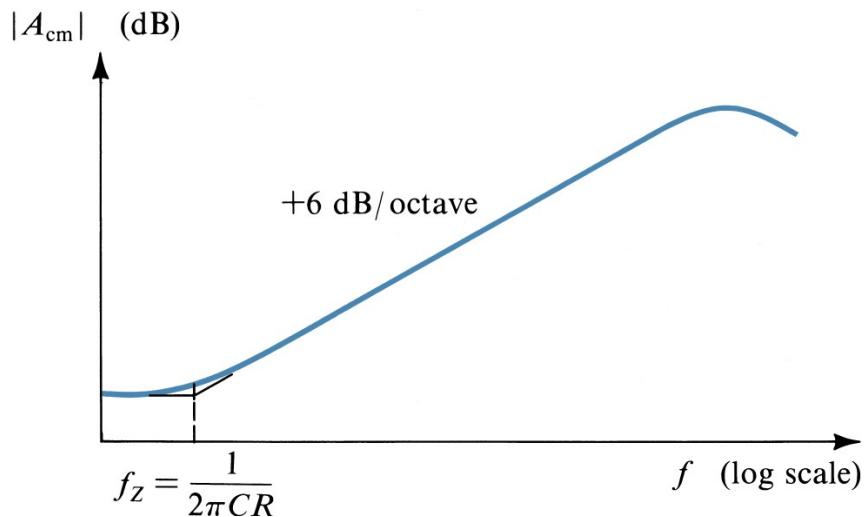
$$\tilde{\tau} = C \cdot (R_L + R_D)$$



**Fig. 7.13** The classical common-emitter amplifier stage. (The nodes are numbered for the purposes of the SPICE simulation in Example 7.9.)



**Fig. 7.14** Equivalent circuit for the amplifier of Fig. 7.13 in the low-frequency band.



**Fig. 7.33** Variation of (a) common-mode gain, (b) differential gain, and (c) common-mode rejection ratio with frequency.

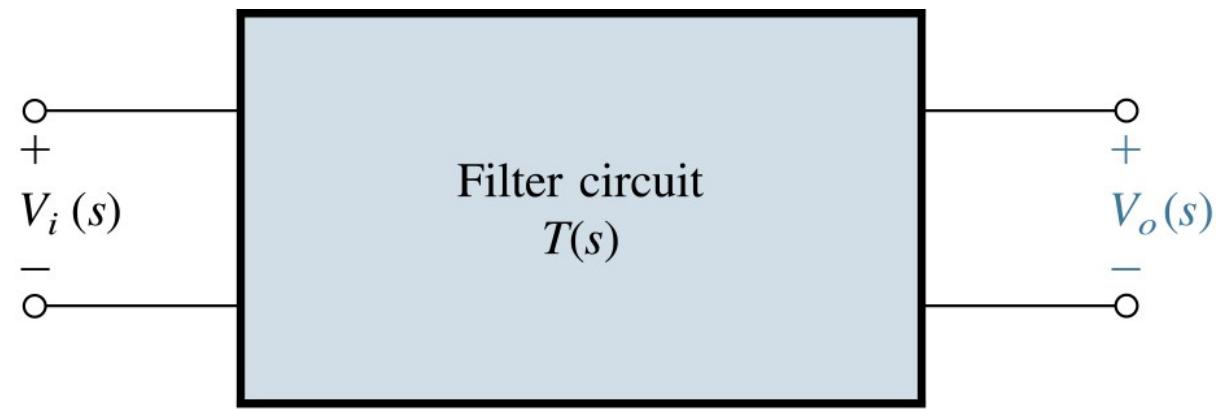
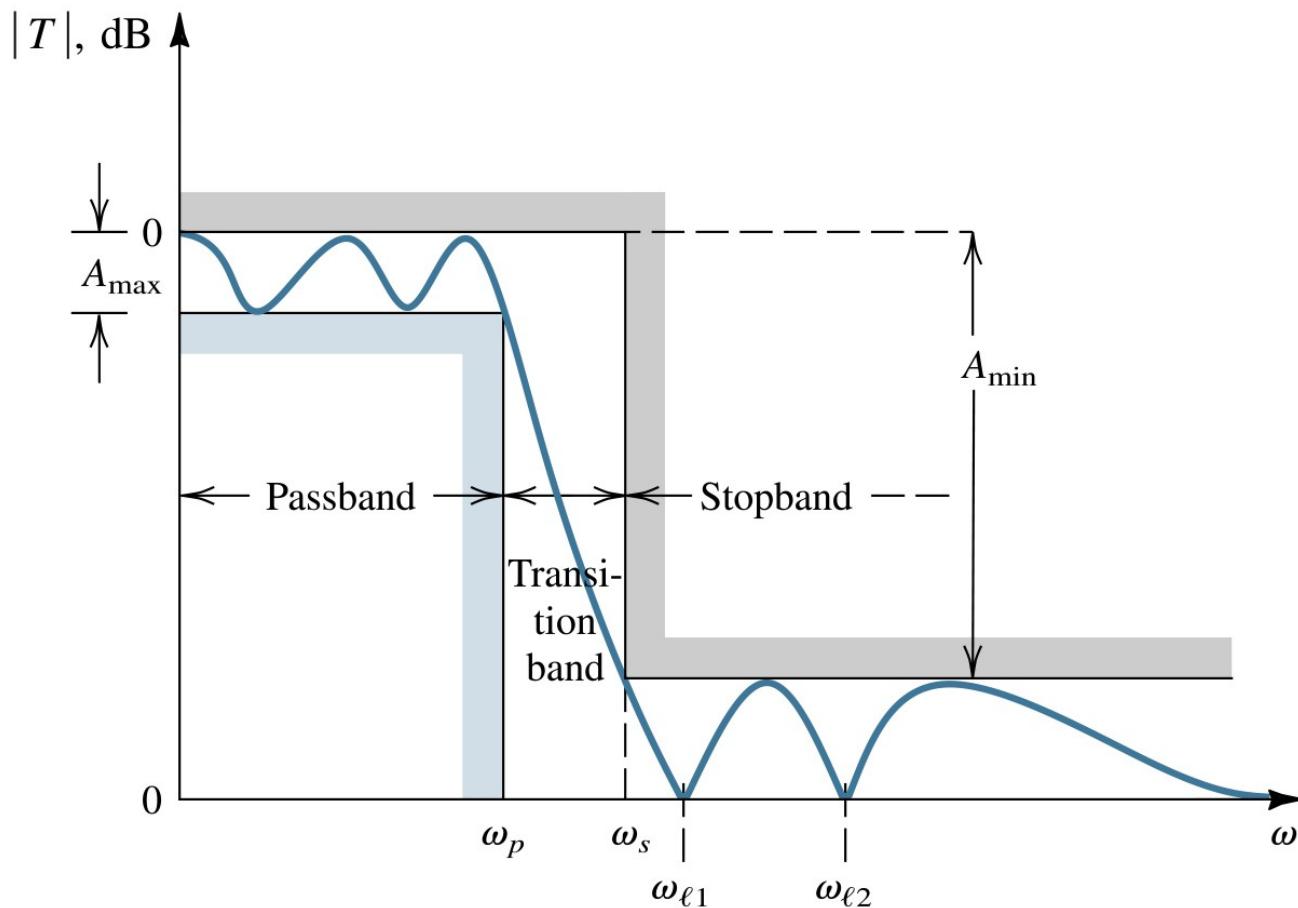
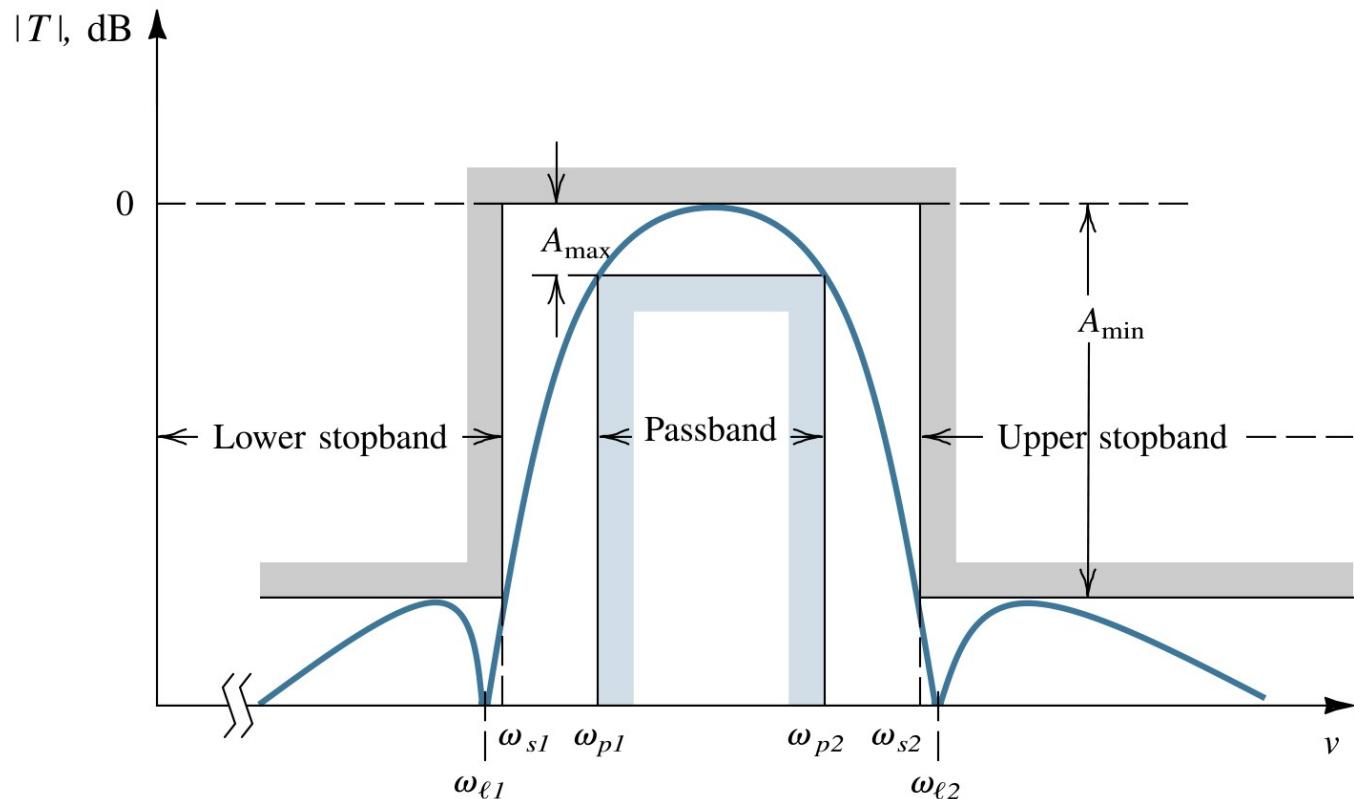


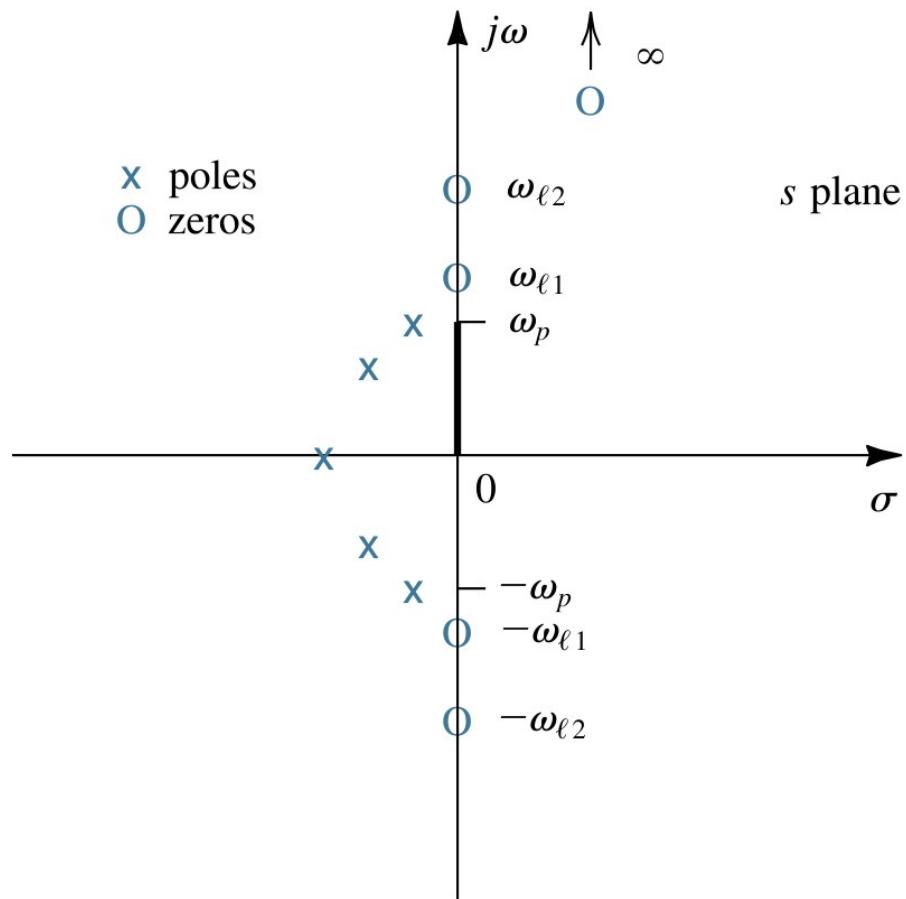
Fig. 11.1



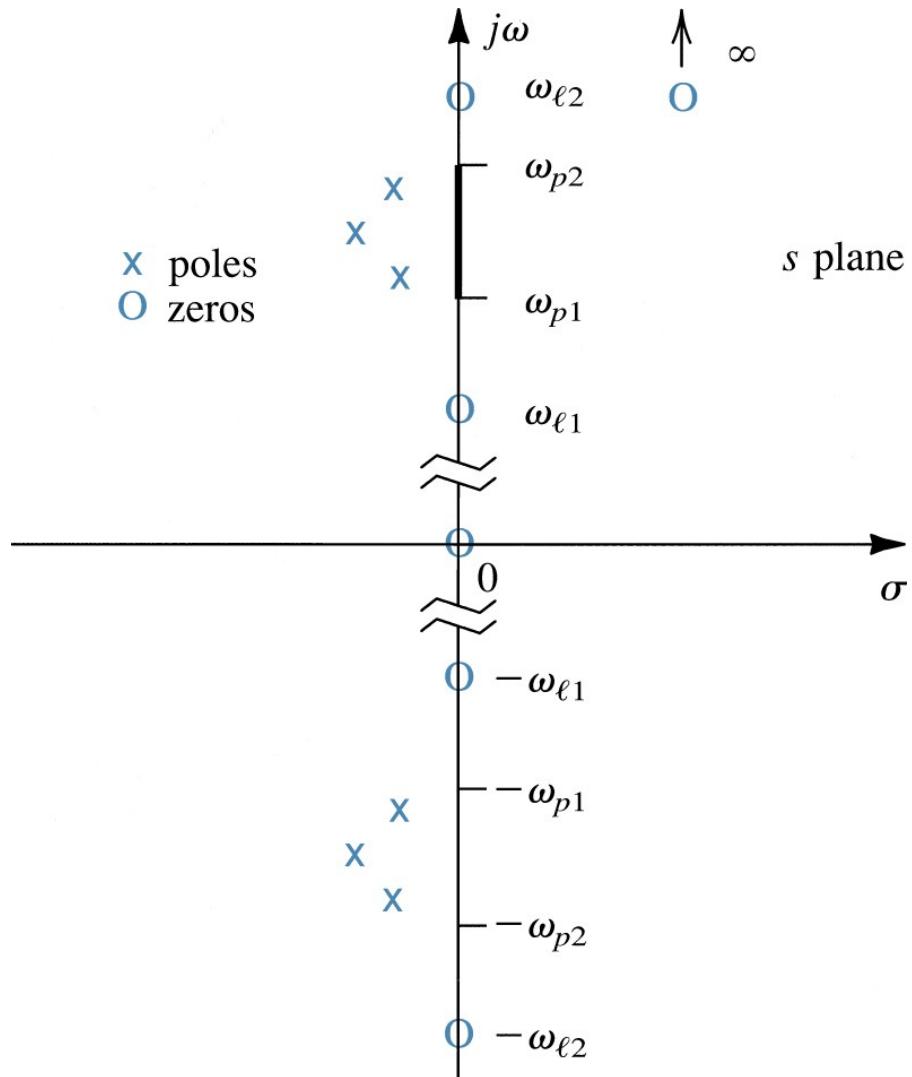
**Fig. 11.3** Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.



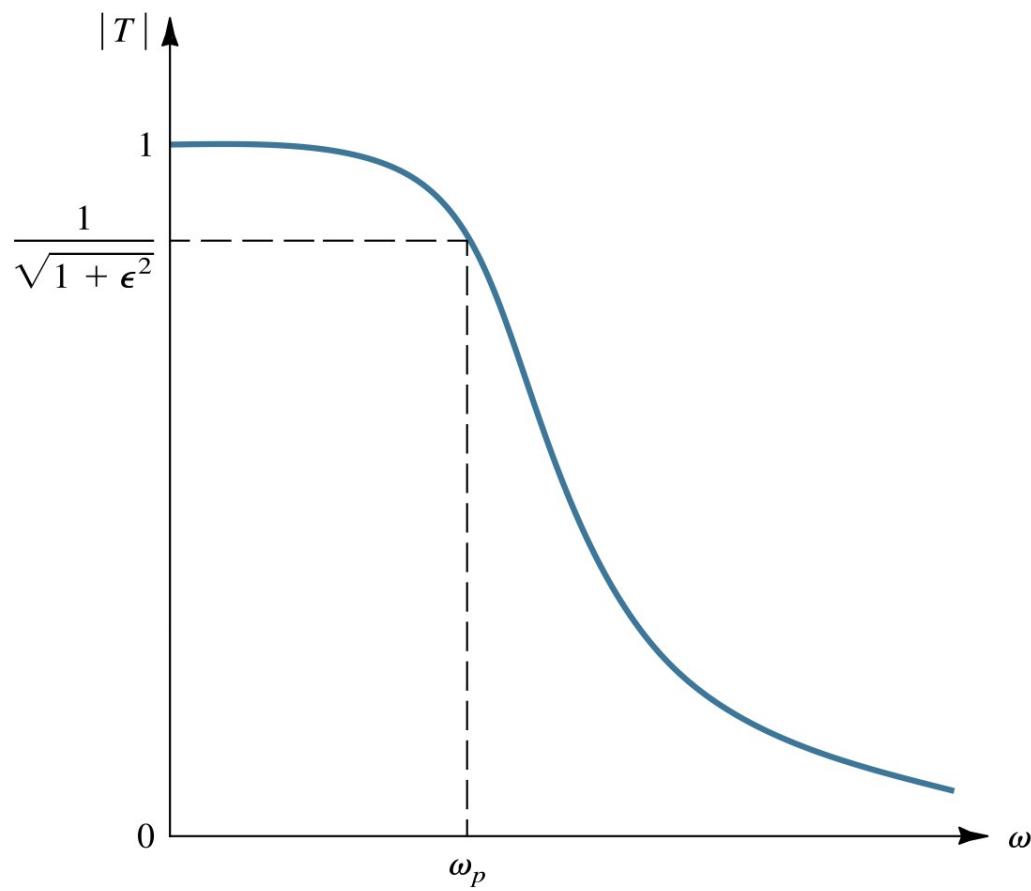
**Fig. 11.4** Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.



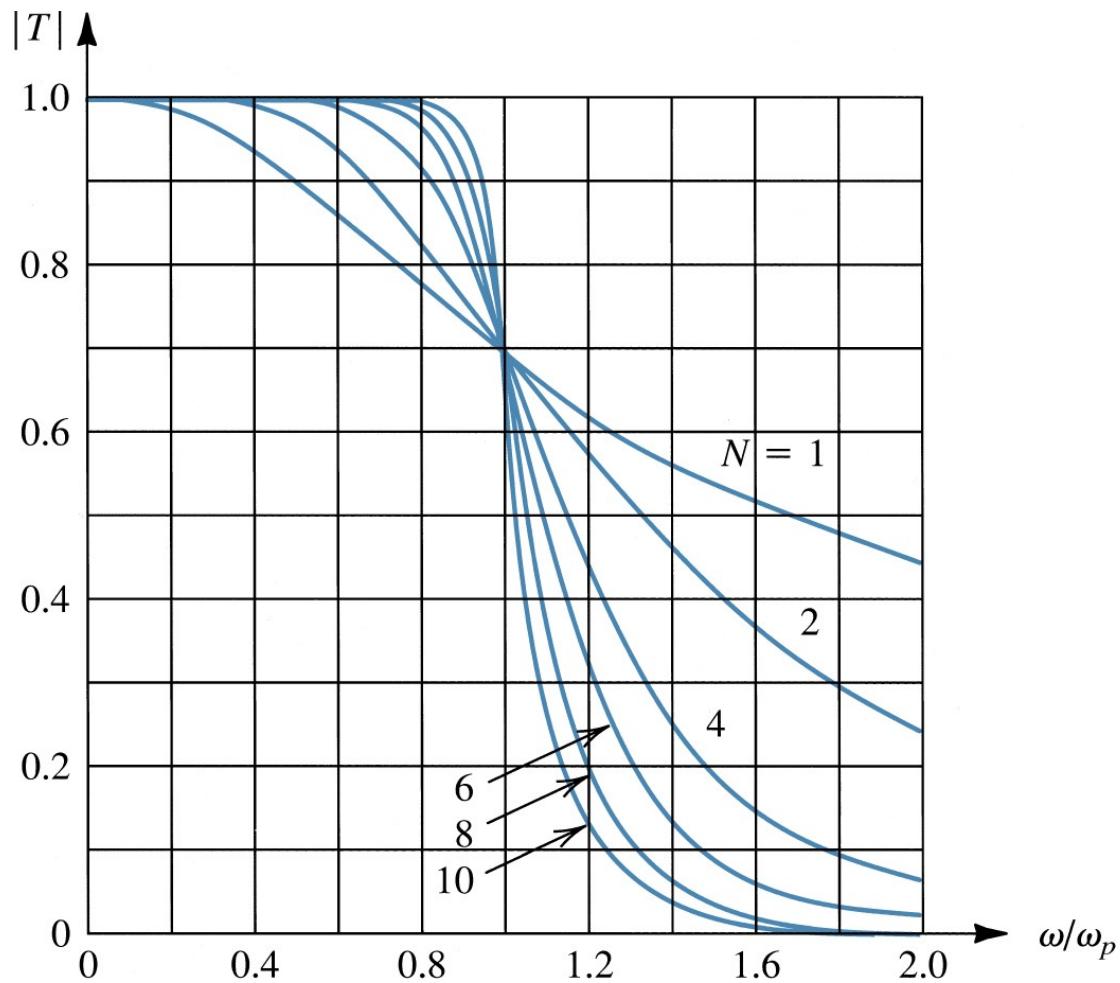
**Fig. 11.5** Pole-zero pattern for the low-pass filter whose transmission is sketched in **Fig. 11.3**. This filter is of the fifth order ( $N = 5$ .)



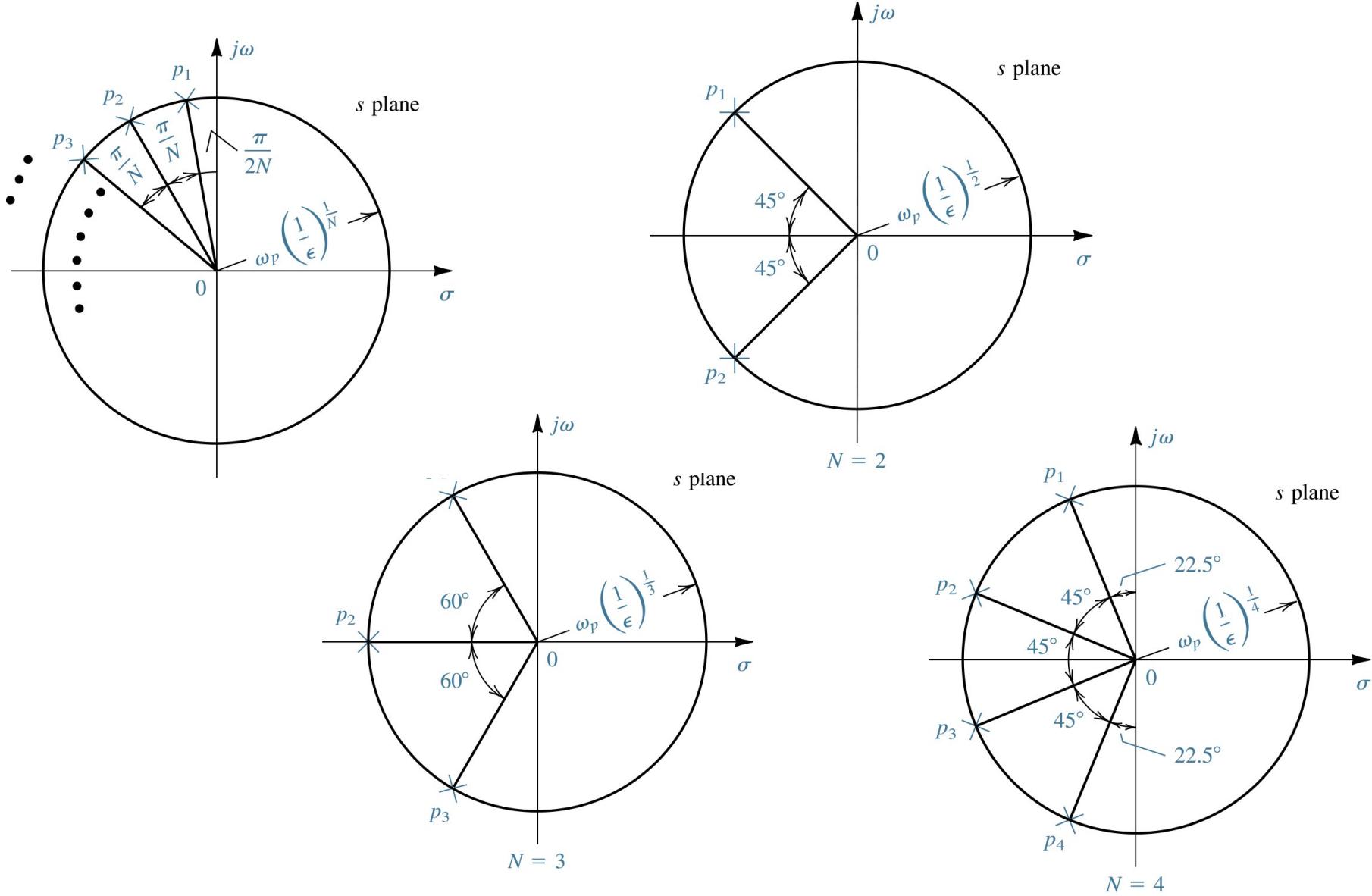
**Fig. 11.6** Pole-zero pattern for the bandpass filter whose transmission is shown in **Fig. 11.4**. This filter is of the sixth order ( $N = 6$ .)



**Fig. 11.8** The magnitude response of a Butterworth filter.



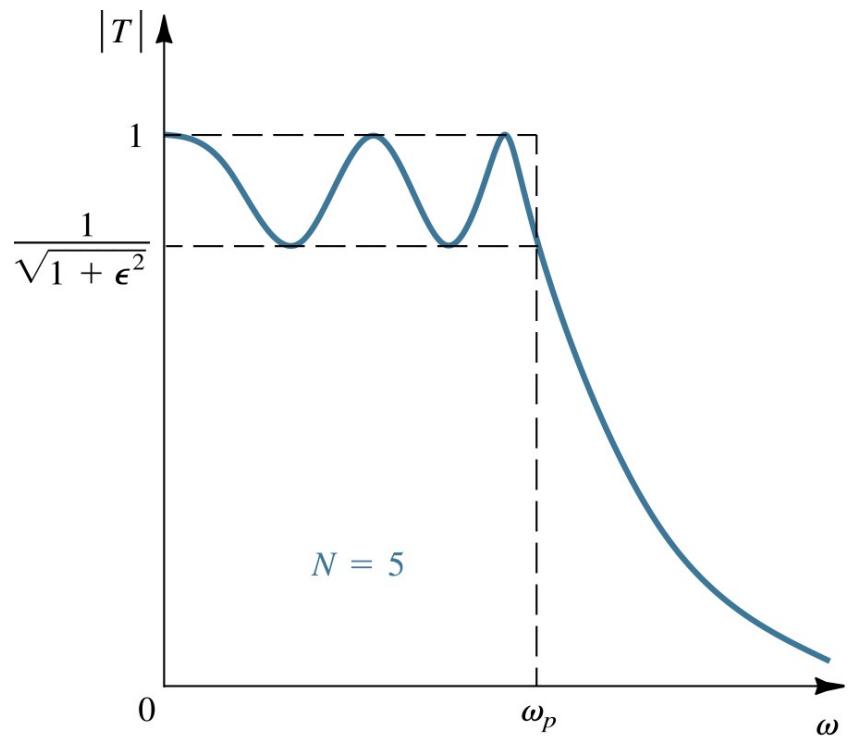
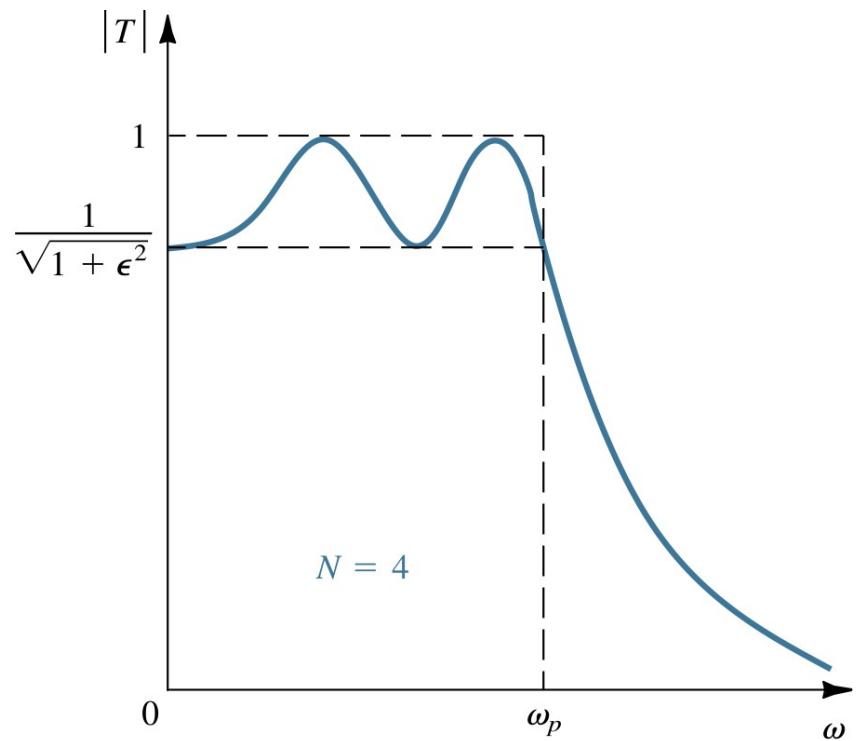
**Fig. 11.9** Magnitude response for Butterworth filters of various order with  $\epsilon = 1$ . Note that as the order increases, the response approaches the ideal brickwall type transmission.



**Fig. 11.10** Graphical construction for determining the poles of a Butterworth filter of order  $N$ . All the poles lie in the left half of the  $s$ -plane on a circle of radius  $\omega_0 = \omega_p (1/\epsilon)^{1/N}$ , where  $\epsilon$  is the passband deviation parameter :

(a) the general case, (b)  $N = 2$ , (c)  $N = 3$ , (d)  $N = 4$ .

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1}$$



**Fig. 11.12** Sketches of the transmission characteristics of a representative even- and odd-order Chebyshev filters.

Filter Type and $T(s)$	$s$ -Plane Singularities	Bode Plot for $ T $	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$		<p>20 log <math>\frac{a_0}{\omega_0}</math></p> <p><math>-20 \frac{\text{dB}}{\text{decade}}</math></p> <p><math>\omega_0</math></p> <p><math>\omega(\text{log})</math></p>	<p><math>R</math></p> <p><math>V_i</math></p> <p><math>V_o</math></p> <p><math>CR = \frac{1}{\omega_0}</math></p> <p>dc gain = 1</p>	<p><math>R_2</math></p> <p><math>R_1</math></p> <p><math>V_i</math></p> <p><math>V_o</math></p> <p><math>CR_2 = \frac{1}{\omega_0}</math></p> <p>dc gain = <math>-\frac{R_2}{R_1}</math></p>
(b) High-Pass (HP) $T(s) = \frac{a_1 s}{s + \omega_0}$		<p>20 log <math> a_1 </math></p> <p><math>+20 \frac{\text{dB}}{\text{decade}}</math></p> <p><math>\omega_0</math></p> <p><math>\omega(\text{log})</math></p>	<p><math>C</math></p> <p><math>V_i</math></p> <p><math>V_o</math></p> <p><math>CR = \frac{1}{\omega_0}</math></p> <p>High-frequency gain = 1</p>	<p><math>R_2</math></p> <p><math>R_1</math></p> <p><math>V_i</math></p> <p><math>V_o</math></p> <p><math>CR_1 = \frac{1}{\omega_0}</math></p> <p>High-frequency gain = <math>-\frac{R_2}{R_1}</math></p>
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$		<p>20 log <math>\frac{a_0}{\omega_0}</math></p> <p>20 log <math>\frac{a_1}{\omega_0}</math></p> <p><math>-20 \frac{\text{dB}}{\text{decade}}</math></p> <p><math>\omega_0</math></p> <p><math>\omega</math></p> <p><math>\frac{a_0}{a_1}(\text{log})</math></p>	<p><math>C_1</math></p> <p><math>R_1</math></p> <p><math>R_2</math></p> <p><math>V_i</math></p> <p><math>V_o</math></p> <p><math>(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}</math></p> <p><math>C_1 R_1 = \frac{a_0}{a_1}</math></p> <p>dc gain = <math>\frac{R_2}{R_1 + R_2}</math></p> <p>HF gain = <math>\frac{C_1}{C_1 + C_2}</math></p>	<p><math>R_2</math></p> <p><math>R_1</math></p> <p><math>C_1</math></p> <p><math>C_2</math></p> <p><math>V_i</math></p> <p><math>V_o</math></p> <p><math>C_2 R_2 = \frac{1}{\omega_0}</math></p> <p><math>C_1 R_1 = \frac{a_1}{a_0}</math></p> <p>dc gain = <math>-\frac{R_2}{R_1}</math></p> <p>HF gain = <math>-\frac{C_1}{C_2}</math></p>

Fig. 11.13 First-order filters.

$T(s)$	Singularities	$ T $ and $\phi$	Passive Realization	Op Amp-RC Realization
$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$ $a_1 > 0$			<p> <math>CR = 1/\omega_0</math>  Flat gain (<math>a_1</math>) = 0.5 </p>	<p> <math>CR = 1/\omega_0</math>  Flat gain (<math>a_1</math>) = 1 </p>

**Fig. 11.14** First-order all-pass filter.

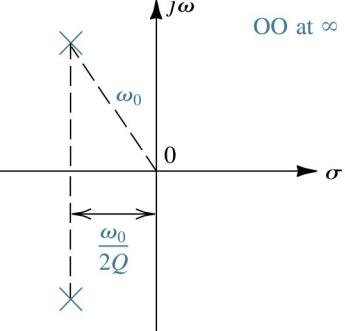
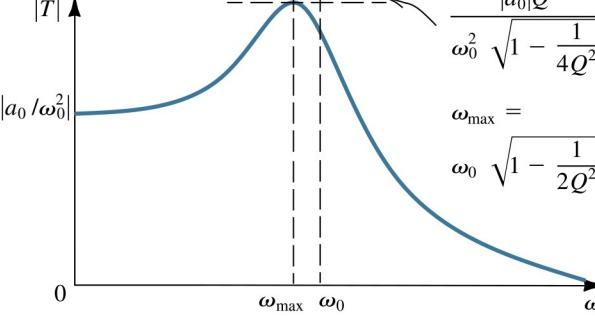
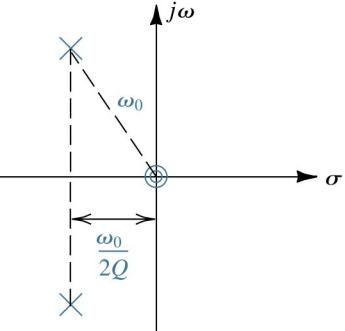
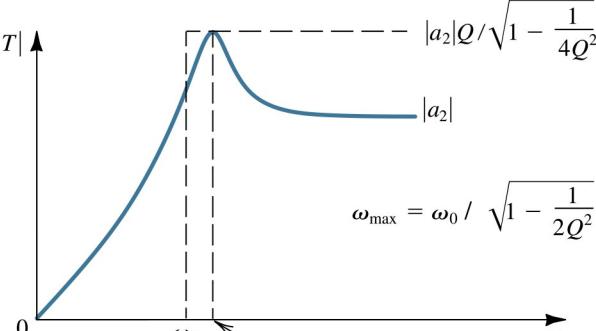
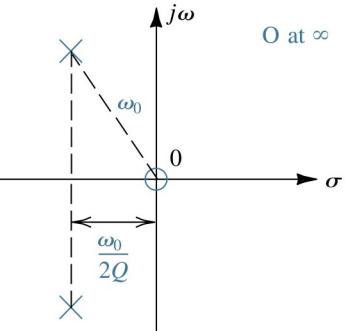
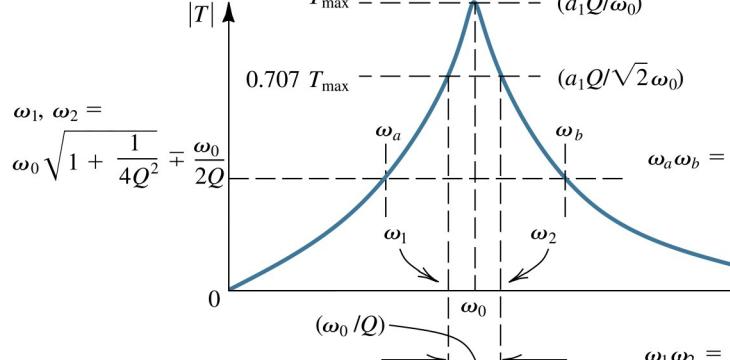
Filter Type and $T(s)$	s-Plane Singularities	$ T $
(a) Low-Pass (LP) $T(s) = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ dc gain = $\frac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain = $a_2$		
(c) Bandpass (BP) $T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain = $a_1 Q / \omega_0$		

Fig. 11.16 Second-order filtering functions.

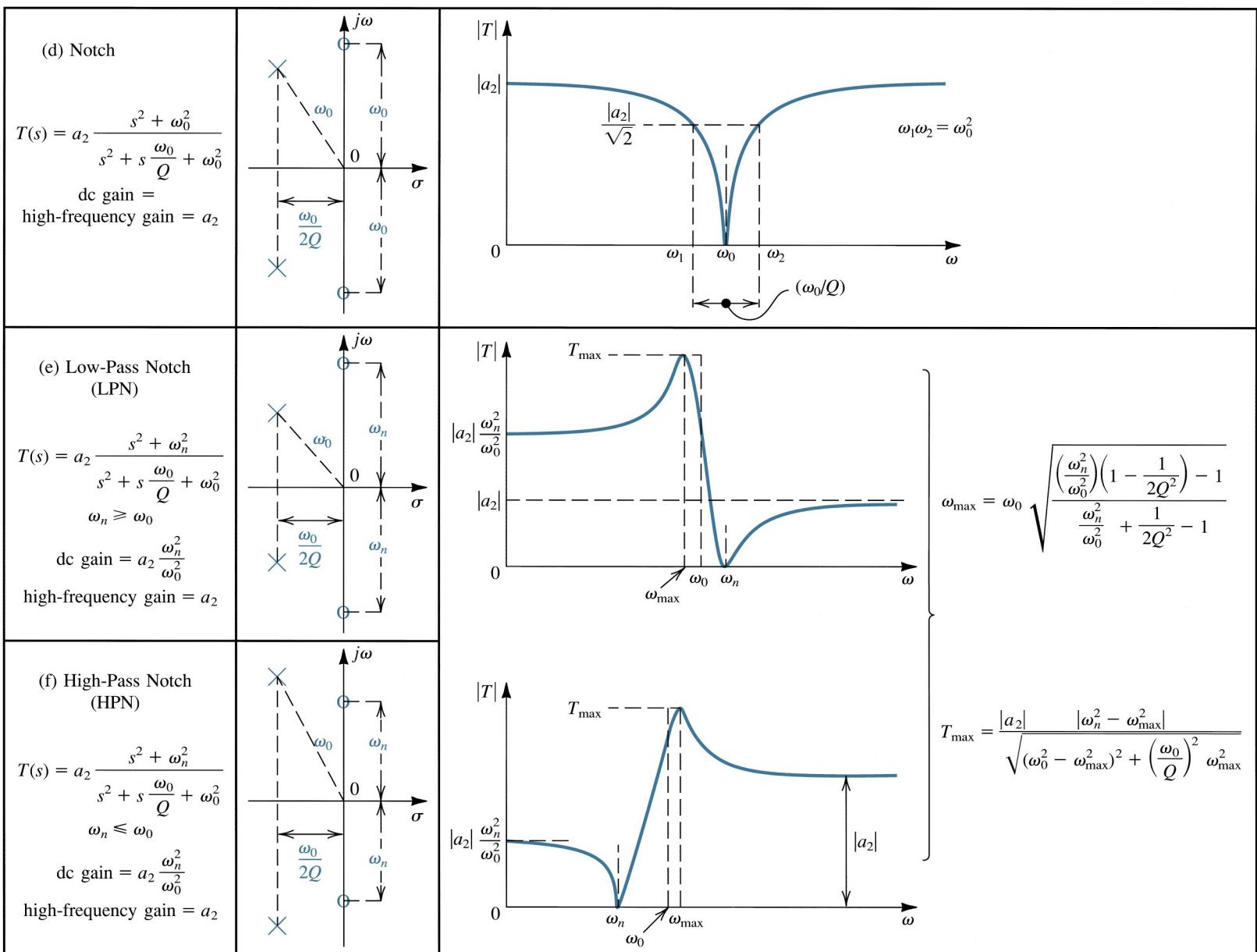
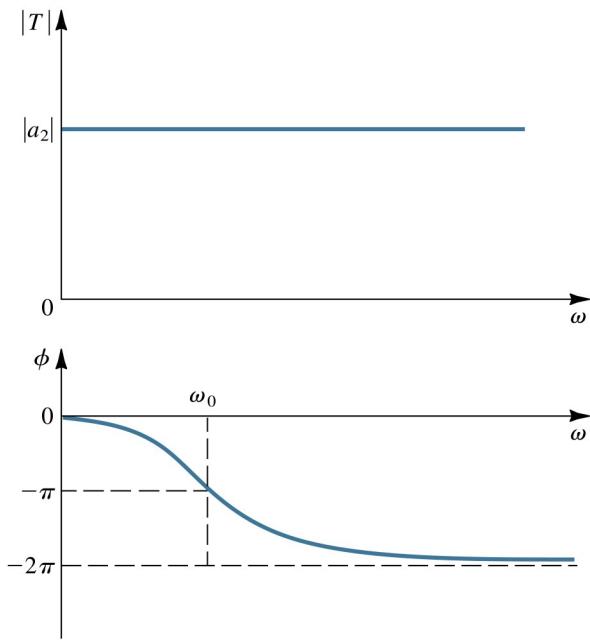
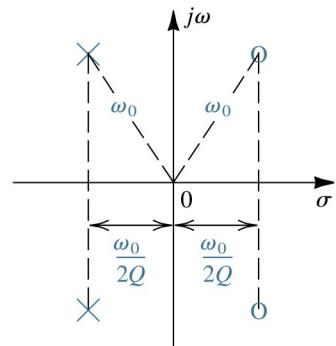


Fig. 11.16 (continued)

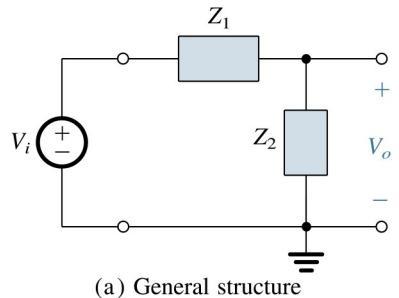
(g) All-Pass  
(AP)

$$T(s) = a_2 \frac{s^2 - s \frac{\omega_0}{Q} + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

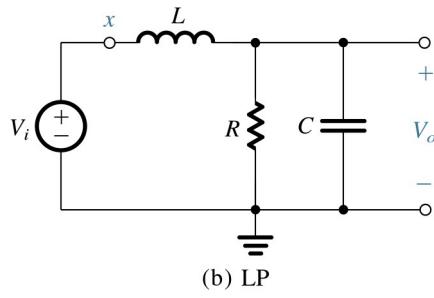
Flat gain =  $a_2$



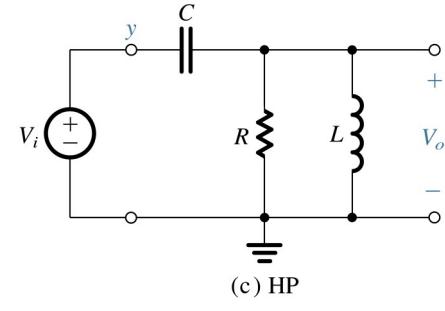
**Fig. 11.16** (continued)



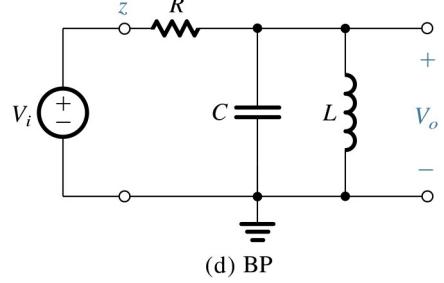
(a) General structure



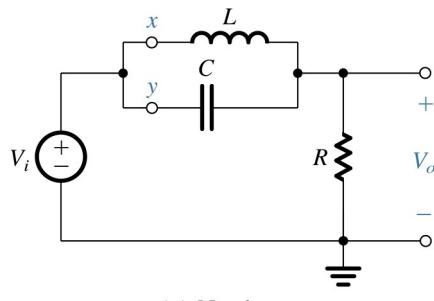
(b) LP



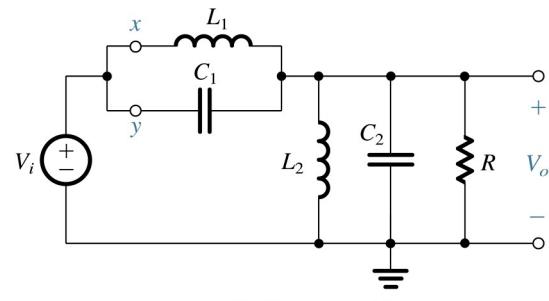
(c) HP



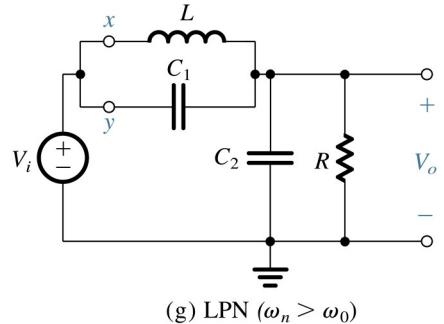
(d) BP



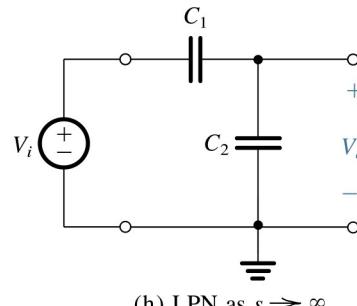
(e) Notch at  $\omega_0$



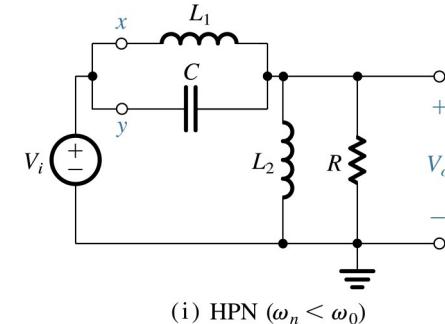
(f) General notch



(g) LPN ( $\omega_n > \omega_0$ )

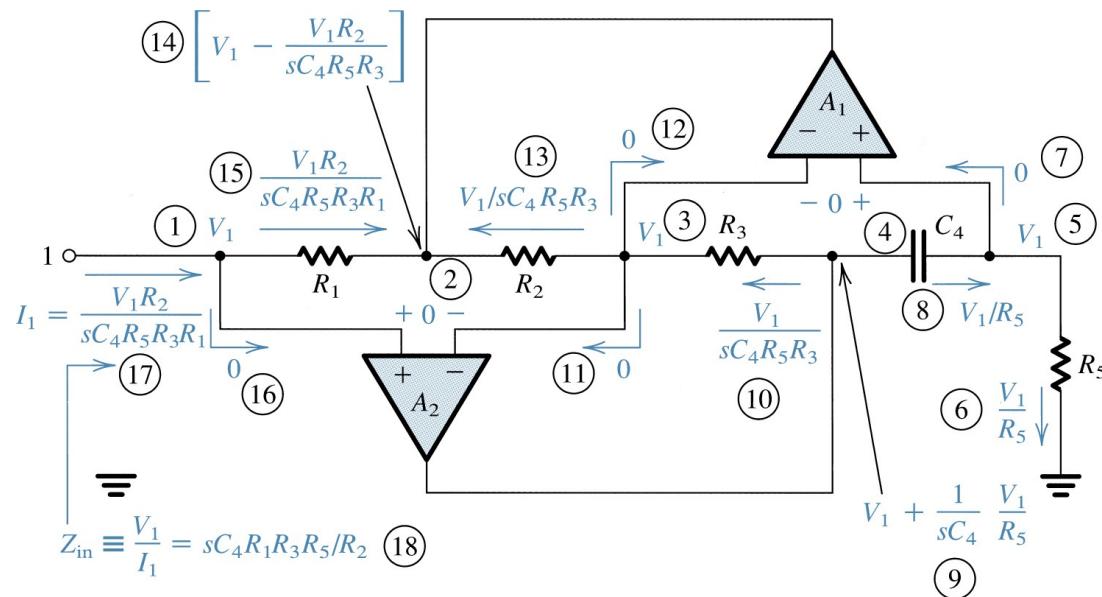
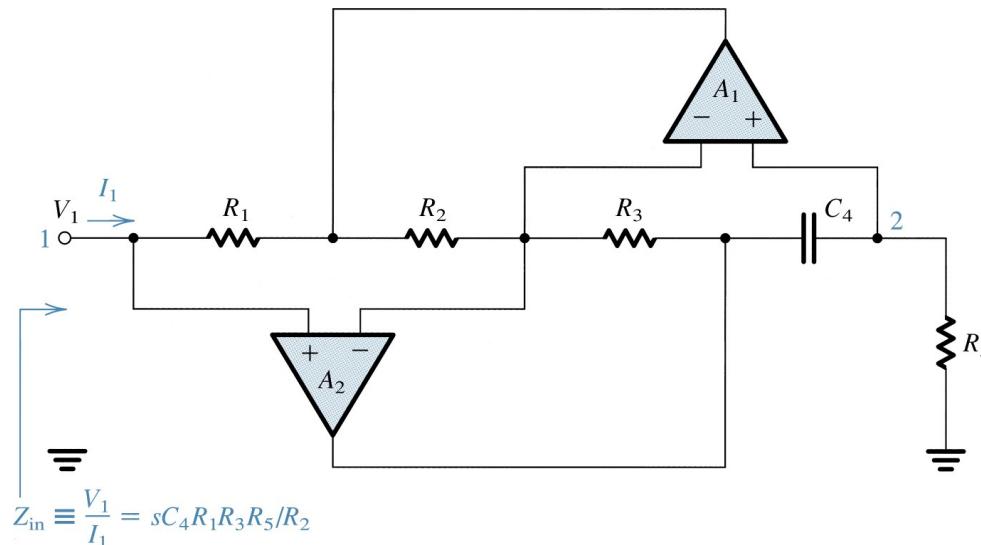


(h) LPN as  $s \rightarrow \infty$

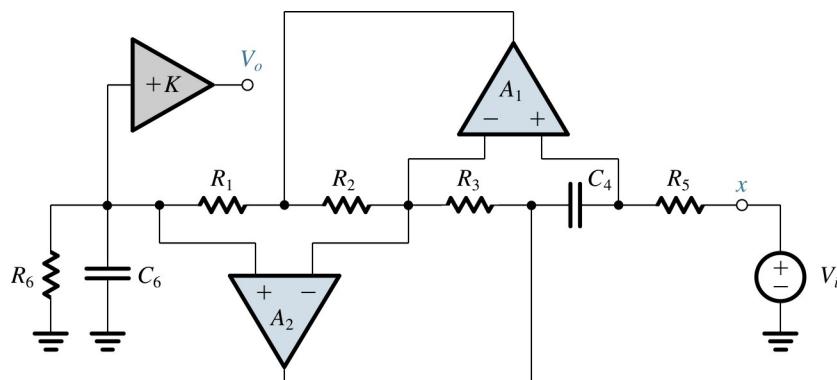


(i) HPN ( $\omega_n < \omega_0$ )

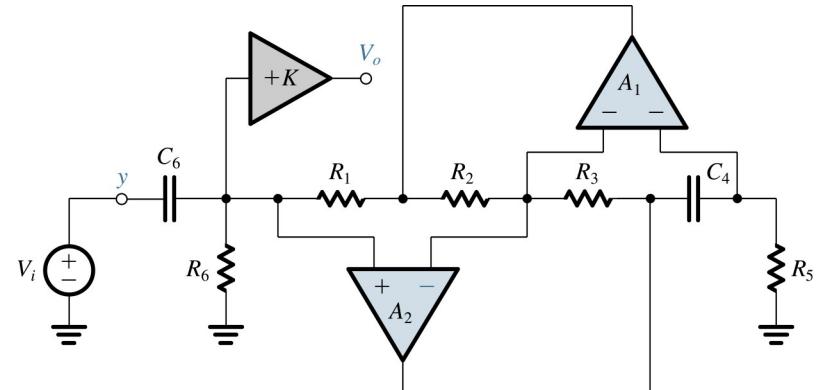
**Fig. 11.18** Realization of various second-order filter functions using the LCR resonator of Fig. 11.17(b): (a) general structure, (b) LP, (c) HP, (d) BP, (e) notch at  $\omega_0$ , (f) general notch, (g) LPN ( $\omega_n \geq \omega_0$ ), (h) LPN as  $s \rightarrow \infty$ , (i) HPN ( $\omega_n < \omega_0$ ).



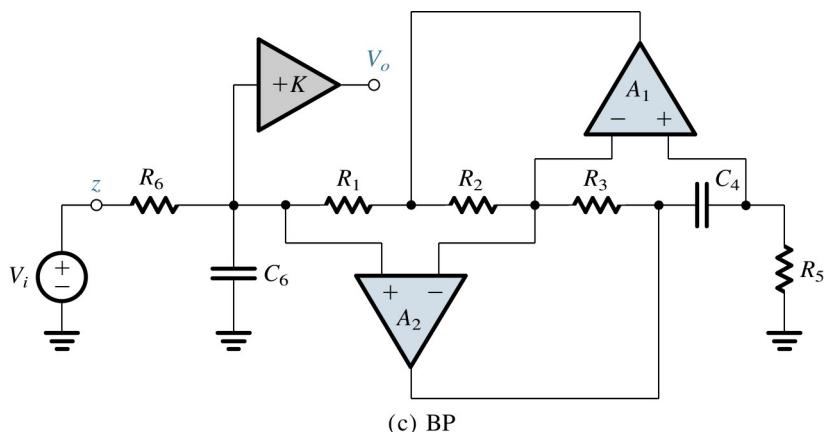
**Fig. 11.20 (a)** The Antoniou inductance-simulation circuit. **(b)** Analysis of the circuit assuming ideal op amps. The order of the analysis steps is indicated by the circled numbers.



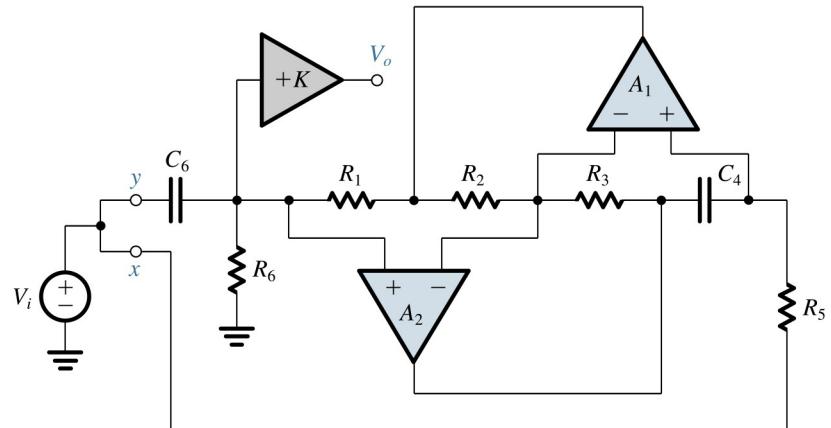
(a) LP



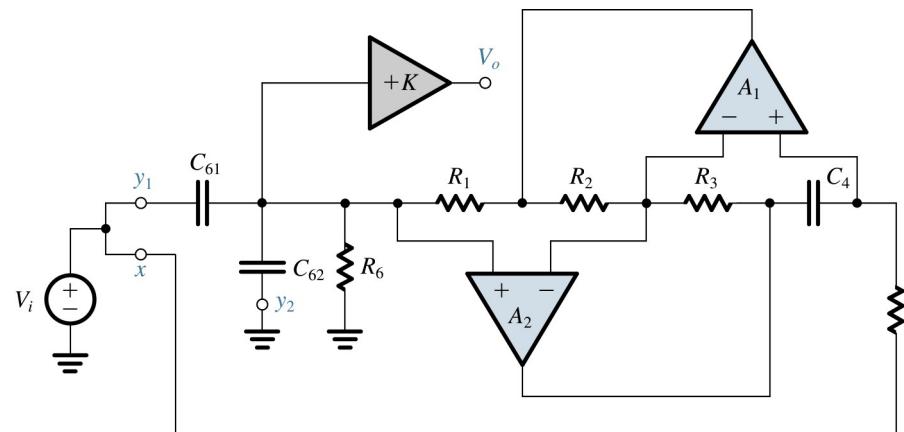
(b) HP



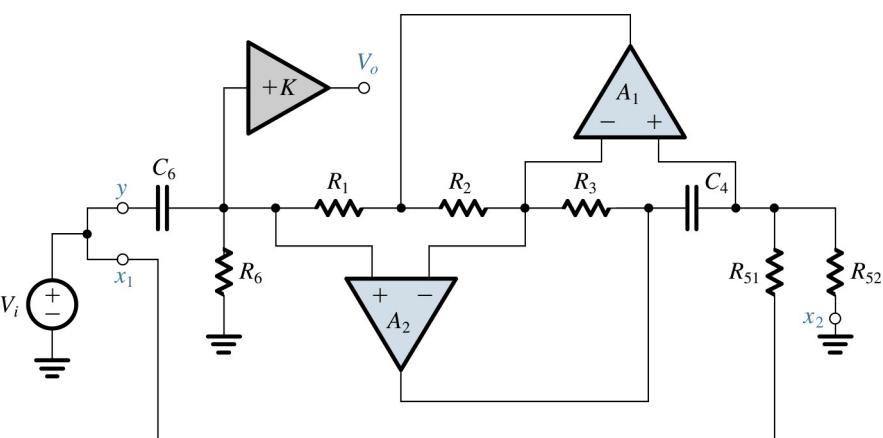
(c) BP

(d) Notch at  $\omega_0$ 

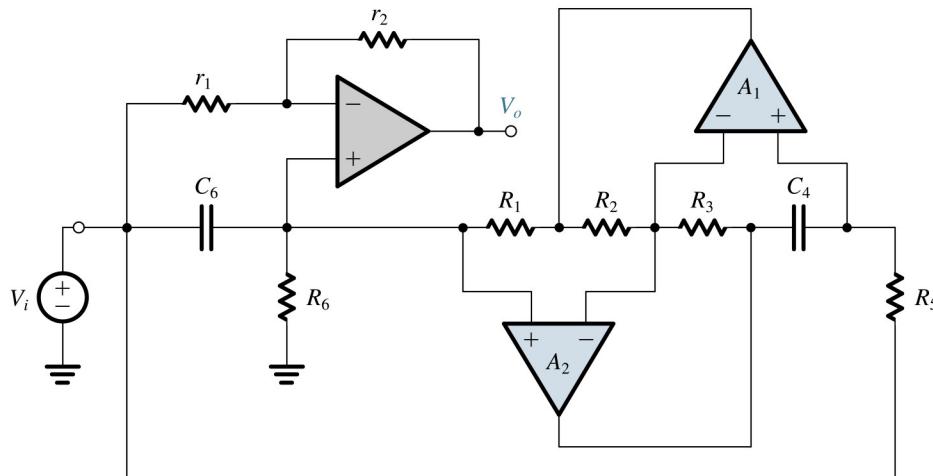
**Fig. 11.22a** Realizations for the various second-order filter functions using the op amp-RC resonator of **Fig. 11.21 (b)**. **(a)** LP; **(b)** HP; **(c)** BP, **(d)** notch at  $\omega_0$ ;



(e) LPN,  $\omega_n \geq \omega_0$

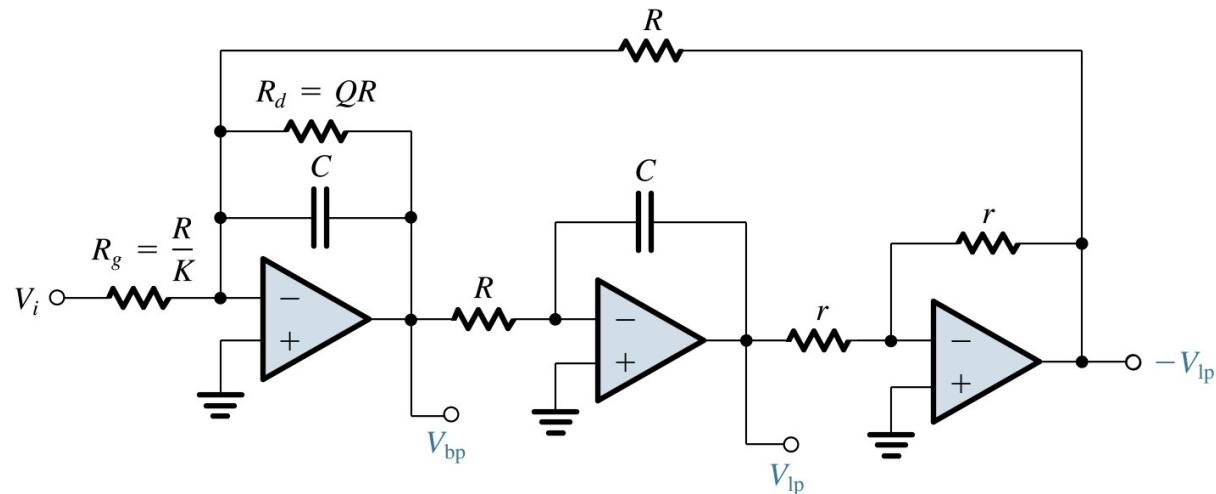
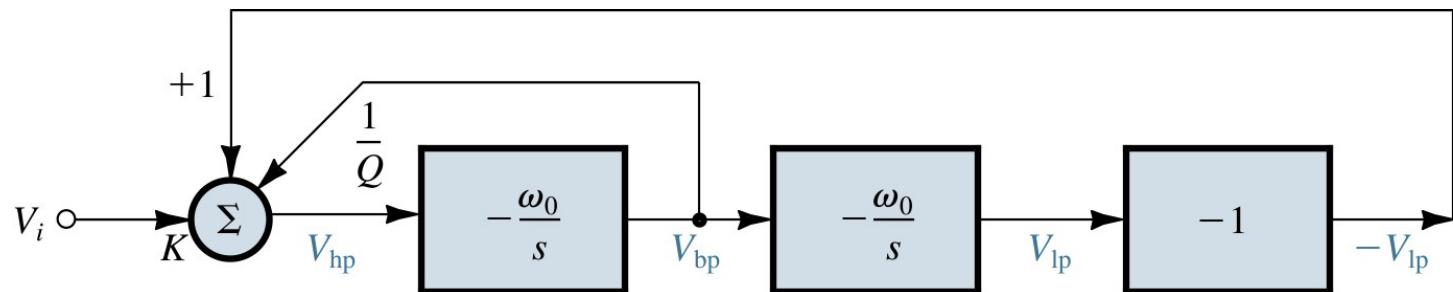


(f) HPN,  $\omega_n \leq \omega_0$

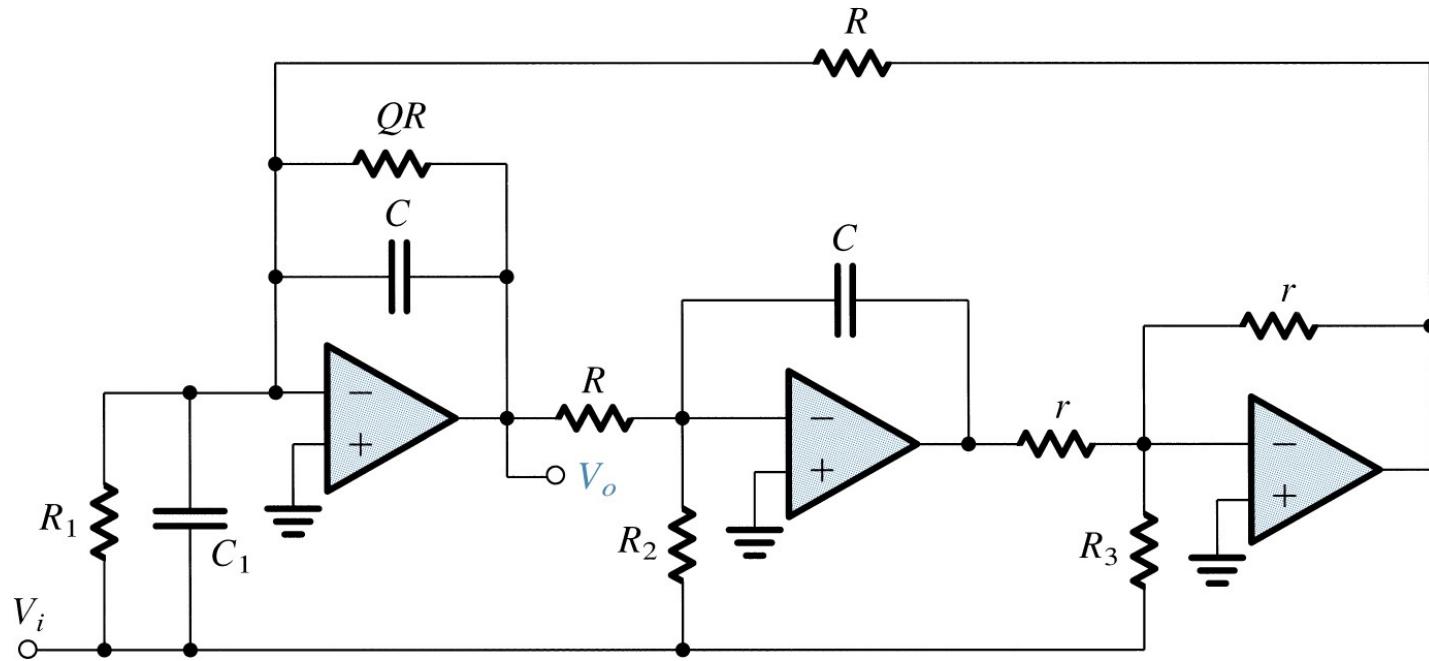


(g) All-pass

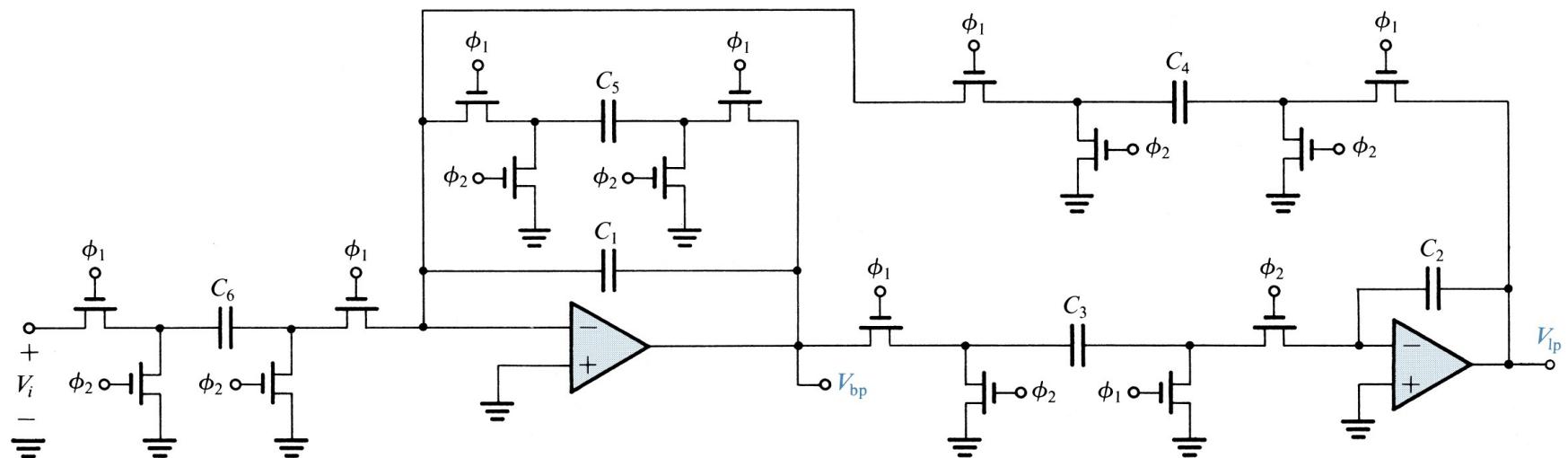
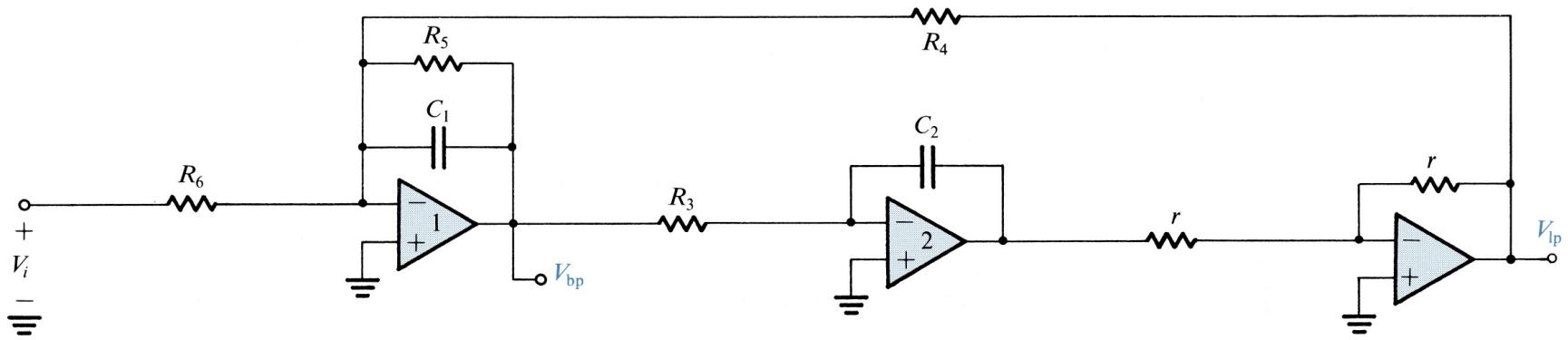
**Fig. 11.22b** (e) LPN,  $\omega_n \geq \omega_0$ ; (f) HPN,  $\omega_n \leq \omega_0$ ; (g) all-pass. The circuits are based on the LCR circuits in **Fig. 11.18**. Design equations are given in **Table 11.1**.



**Fig. 11.25** Derivation of an alternative two-integrator-loop biquad in which all op amps are used in a single-ended fashion. The resulting circuit in (b) is known as the Tow-Thomas biquad.



**Fig. 11.26** The Tow-Thomas biquad with feedforward. The transfer function of Eq. (11.68) is realized by feeding the input signal through appropriate components to the inputs of the three op amps. This circuit can realize all special second-order functions. The design equations are given in **Table 11.2**.



**Fig. 11.37** A two-integrator-loop active-RC biquad and its switched-capacitor counterpart.